

Bitcoin: The New Virtual Gold?

An investigation into the diversification properties of Bitcoin
within a South African portfolio

By

Brett Hope Robertson

HPRBRE001

SUBMITTED TO THE UNIVERSITY OF CAPE TOWN

In partial fulfilment of the requirements for the degree Master of
Commerce specialising in Finance in the field of Investment
Management

Faculty of Commerce

UNIVERSITY OF CAPE TOWN

23 July 2018

Supervisor: Associate Professor Francois Toerien

Department of Finance and Tax

The copyright of this thesis vests in the author. No quotation from it or information derived from it is to be published without full acknowledgement of the source. The thesis is to be used for private study or non-commercial research purposes only.

Published by the University of Cape Town (UCT) in terms of the non-exclusive license granted to UCT by the author.

Plagiarism declaration

I, *Brett Hope Robertson*, hereby declare that the work on which this dissertation/thesis is based is my original work (except where acknowledgements indicate otherwise) and that neither the whole work nor any part of it has been, is being, or is to be submitted for another degree in this or any other university.

I empower the university to reproduce for the purpose of research either the whole or any portion of the contents in any manner whatsoever.

Signature:

Date: 23/07/2018

Abstract

Using August 2010 to February 2018 as the sample period, this study investigates, from the perspective of a South African investor, the portfolio diversification and optimisation abilities and asset allocation effects of including Bitcoin in a portfolio, and compares this with physical gold. In particular the study investigates: (i) key statistics, returns and correlations between Bitcoin's returns and those of the components of a standard South African base portfolio and gold bullion; (ii) the risk-return efficiency enhancements and asset allocation effects (weightings) from the inclusion of Bitcoin and gold bullion under different portfolio frameworks¹; (iii) the cumulative returns of the optimal portfolios over the investment period; and (iv) the efficient frontier shifts from the inclusion of Bitcoin and gold bullion within the context of different portfolio frameworks.

Because of Bitcoin's non-normal return distribution and extremely high returns, a quantitative mean-semivariance based portfolio optimisation approach is followed, using index proxies² for the traditional six portfolio assets included (SA equity, SA bonds SA listed property, SA cash/money market, international equity and international cash).

The study finds that, for a South African portfolio, Bitcoin (i) enhances the risk-return efficiency (measured i.t.o. the Sortino ratio) and shifts the efficient frontier up and outwards under every portfolio framework, and (ii) that it is far superior to physical gold in this regard under all portfolio frameworks, including within the asset weighting constraints imposed by South African legislation (Regulation 28 of the Pension Fund Act). In addition, at weightings of 5% or lower, Bitcoin not only enhances the portfolio's risk-return profile, but also further reduces the outright risk of the optimal portfolio. In addition, it is found that due to its negative correlation with SA bonds and SA cash, Bitcoin's inclusion in a portfolio increases the weighting of these assets, and reduces the allocation to SA and global equity.

The above results indicate that South African investors with a higher risk tolerance should consider allocating a small part of their portfolio to Bitcoin, and that South African regulatory authorities should make provision for cryptocurrencies when drafting investment legislation (and, in particular, should consider clarifying its status within Regulation 28).

¹ Long-only unconstrained, long-only constrained, equally-weighted, and +100%/-100%

² The proxies used were as follows: the Bitcoin Price Index, the JSE All Bond Index Total Return (ABLITR), the FTSE/JSE All Share Total Return Index (JALSHR), the South African Listed Property Index (JSAPY), the SA Short Term Fixed Interest Rate Composite Index (STeFI), the Bloomberg Global Developed Sovereign Bond Index (5 - 7 year), the MSCI World Index, and the London Gold Bullion closing price.

Table of Contents

Chapter 1: Introduction and Background.....	1
1.1 Research problem statement	3
1.2 Aims and Objectives	3
1.3 Research question	4
1.4 Study contribution.....	4
1.5 Thesis structure	5
Chapter 2: Overview: Bitcoin & South African financial markets.....	6
2.1 Bitcoin.....	6
2.2 South African financial markets and asset classes.....	10
2.3 Conclusion	11
Chapter 3: Literature review	13
3.1 Diversification theory and methods	13
3.2 Bitcoin as an asset class	15
3.3 Bitcoin diversification: empirical findings and evidence	16
3.4 Gold as a diversification asset: empirical evidence and findings	23
Chapter 4: Data	27
Chapter 5: Methodology	30
5.1 The Mean-Semivariance Framework.....	30
5.2 Investment and optimisation process	34
5.3 Portfolio Strategy	39
5.3.1. Regulation 28	39
5.3.2. Portfolio framework.....	40
5.3.2.1 Theoretical Frameworks	40
5.3.2.2 Practical Framework	41
Chapter 6: Results and analysis	43
6.1 Explanatory and descriptive data analysis	43
6.2 Portfolio results and analysis	48
6.2.1 Analysis into portfolio optimisation under different frameworks	48
6.2.2 Analysis of the efficient frontiers under different frameworks	55
6.2.3 Direct comparison of Bitcoin and gold under different frameworks.....	59
Chapter 7: Conclusion.....	66

7.1 Review of study	66
7.2 Limitations of study	67
7.3 Further research	68
References.....	69
Appendices.....	74

List of Tables

Table 3.1 - Briere, Oosterlinck, and Szafarz (2015) correlation matrix results.	18
Table 3.2 – Eisl, Gasser and Weinmayer (2015). Correlation Matrix.	19
Table 3.3 – Eisl et al. (2015). Detailed results of each portfolio optimisation framework.	21
Table 3.4 - Yermack’s correlation matrix results	22
Table 3.5 - Baur and McDermott (2010) GRACH model daily return results	25
Table 4.1 – Data and sources of data used in this study	27
Table 6.1: Descriptive statistics	43
Table 6.2: Correlation Matrix	45
Table 6.2: Portfolio optimisation results.....	49
Table 6.3: Portfolio optimisation results.....	60

List of Figures

Figure 1.1- Bitcoin USD price versus the USD/rand cross rate	9
Figure 3.1 - Briere, Oosterlinck and Szafarz (2015) mean-variance efficient frontier.....	20
Figure 3.2 – Chua et al’s common stock portfolio with gold bullion.	24
Figure 5.1: Bitcoin Histogram	30
Figure 6.1: Asset classes’ cumulative monthly returns	46
Figure 6.2: Asset classes cumulative monthly returns.....	47
Figure 6.3: Long-only constrained portfolios cumulated returns	53
Figure 6.4: All Bitcoin inclusive portfolios’ cumulated returns.....	54
Figure 6.5: The efficient frontier of unconstrained long-only portfolios.....	55
Figure 6.6: The efficient frontier of constrained long-only portfolios.....	57
Figure 6.8: The efficient frontier of constrained long-only portfolios.....	63
Figure 6.9: The efficient frontier of constrained long-only portfolios.....	64

Chapter 1: Introduction and Background

The recent boom in the price of Bitcoin has prompted investors to explore the possible investment opportunities of this cryptocurrency. Bitcoin is a digital, unregulated and partially anonymous currency or form of payment which is not backed by any government or legal entity (Grinberg, 2012). Bitcoin has been dubbed the ‘Virtual Gold’ and the gold for millennials (Burniske & White, 2016), and could potentially be a new portfolio diversifier and ‘safe-haven’ option for South African investors. A ‘safe-haven’ asset is defined as one that is uncorrelated or negatively correlated with other asset classes or portfolios in times of market uncertainties or turmoil (Baur & Lucey, 2010), and given that some researchers have found low to negative correlations between Bitcoin returns and those of multiple assets classes in US portfolios³, it is worth testing whether Bitcoin could fulfil a similar role in the South African context. Furthermore, as the returns on gold have historically been perceived as having a low correlation with financial asset classes, especially in times of crisis or uncertainty, it would be interesting to test whether Bitcoin or gold has better diversification benefits within South African portfolios.

This study therefore seeks to:

- a. Investigate the portfolio diversification and optimisation abilities and asset allocation effects of the inclusion of Bitcoin in a portfolio from the perspective of a South Africa investor, using 2010-2017 as the sample period.
- b. Compare Bitcoin to that of gold bullion, in terms of its abilities as a portfolio diversifier, and to test whether its abilities as a diversifier and portfolio optimiser outweigh that of gold bullion.

This study assesses the diversification characteristics of Bitcoin through four main statistical and analytical methods, as follows:

³ See, for example, Baur, Lee and Hong (2015); Yermack, (2013); Burniske and White, (2016); Wu and Pandey (2014); Brière, Oosterlinck and Szafarz (2015); and Eisl, Gasser and Weinmayer (2015)

- i) A full analysis of key statistics, returns and correlations between Bitcoin's returns and return elements of a standard South African base portfolio, consisting of domestic equity, bonds, real estate, cash, foreign equities and foreign bonds; as well as gold.
- ii) An evaluation and comparison on the risk-return efficiency enhancements and asset allocation effects (weightings) from the inclusion of Bitcoin *and* gold bullion under different portfolio frameworks⁴.
- iii) A comparison of the cumulative returns of the optimal portfolios over the investment period.
- iv) An evaluation of the efficient frontier shifts (the minimum semivariance, optimal and maximum return portfolios) from the inclusion of Bitcoin and gold bullion, within the context of different portfolio frameworks.

This study sets out to determine whether Bitcoin can improve the risk-return profile of an already diversified portfolio, and shift the efficient frontier in a way that makes it a diversifying option for a South African investor. In addition, an investigation to compare the diversification abilities of Bitcoin to that of gold bullion under practical South African investment constraints is conducted.

Correlation refers to the degree to which two securities/variables move together. Investigating the correlation between Bitcoin's returns and that of a standard South African portfolio is essential in identifying the diversification ability of Bitcoin within a South African portfolio. The risk-return measure of an optimal portfolio indicates to investors the risk-adjusted return of the portfolio, and is therefore a comparable measure when assessing relative performance of optimal portfolios. The Sharpe and Sortino ratios have become industry wide ratios to assess performance. Bitcoin returns are not normally distributed, which renders the commonly known Sharpe ratio unsuitable. This study therefore uses the mean-semivariance portfolio optimisation approach and the Sortino ratio, and finds weightings under the different portfolio frameworks mentioned above, using the mean-semivariance optimisation set out by Markowitz

⁴ Long-only unconstrained, long-only constrained, equally-weighted, and +100%/-100%

(1959) and Estrada (2007). A mean-semivariance framework allows for Bitcoin's non-normal distributed returns to be interpreted through only assessing the downside risk. From the existing South African portfolio, this framework can determine whether the risk-return profile of the portfolio can be improved by including another investment (Bitcoin) under different portfolio frameworks.

1.1 Research problem statement

Cryptocurrencies (and Bitcoin in particular) represent a new asset class available to South African investors. However, little is known about the effects of including Bitcoin in South African investment portfolios, and whether its impact compares to the inclusion of gold, which traditionally has been seen as low correlation diversification asset. It is these problems that this study seeks to address.

1.2 Aims and Objectives

The aim of this study is to test the enhancing portfolio diversification abilities when including Bitcoin under different portfolio frameworks from a South African investor's point of view. This study will also test whether Bitcoin enhances the efficient frontier and risk-return profile under different portfolio frameworks when compared to that of the traditional gold diversifier.

This study will achieve the abovementioned aims by including Bitcoin under different portfolio frameworks to assess if the inclusion of Bitcoin in a well-diversified South African base portfolio (domestic equity, bonds, real estate, cash, foreign bonds and equity) will enhance the efficient frontier and find a new optimal portfolio under mean-semivariance optimisation. This will first be analysed by adding Bitcoin to a South African base portfolio that already includes the gold bullions assets and hence, already includes the diversification benefits of gold bullion. By adding Bitcoin over and above gold bullion the study can analyse the further diversification enhancements. Further, a direct comparison between Bitcoin and gold bullion will be done by adding each of these separately to the base portfolio, to compare their individual diversification abilities, subject to South Africa's Regulation 28.

The study will analyse the diversification effects through a comparative correlation matrix, efficient frontier shifts, optimal risk-return enhancements and the asset allocation effect on optimal weightings under different portfolio frameworks.

1.3 Research question

The following research questions are addressed in this study:

- Does Bitcoin enhance the efficient frontier and, in turn, the risk-return profile of a well-diversified South African portfolio?
- Does Bitcoin affect the asset allocation of a well-diversified South African portfolio?
- Does Bitcoin have better diversification abilities than gold bullion within a South African portfolio?

1.4 Study contribution

There exists very limited academic research on cryptocurrencies as diversification assets within a portfolio context, as most academic papers on cryptocurrencies are dedicated to legal issues such as money laundering and income tax. Furthermore, previous academic research on the portfolio diversification abilities of cryptocurrencies were done within developed market contexts (mainly the United States⁵), and not in an emerging market context such as South Africa.

Emerging markets are more prone to experience shocks through exchange rate devaluations, political crisis and regulatory changes (Bakaret et al., 1998). High emerging market volatility compared to developed markets stems from factors such as a greater susceptibility to external price shocks, currency swings, and domestic policy instability (Amadeo, 2017). Similarly, extended research has shown that the equity return premium in emerging markets are higher than that of developed markets (Salomons & Grootveld, 2003). A study by de Groot and Zhou (2015) indicates that, based on long-run equity returns and volatility, emerging market equity Sharpe ratios are higher than those of developed markets. Furthermore, South Africa's equity market is highly concentrated, with over 80% of the market capitalisation coming from the top 40 companies (Courtney Capital, 2017), and this concentration adds additional risk to market

⁵ See, for example, Baur, Lee and Hong (2015); Yermack, (2013); Burniske and White, (2016); Wu and Pandey (2014); Brière, Oosterlinck and Szafarz (2015); and Eisl, Gasser and Weinmayer (2015)

portfolio diversification.

In view of the above context, Bitcoin may have different portfolio diversification benefits in an emerging market context compared to those found within developed markets, and this may be of interest to both retail and institutional investors.

1.5 Organisation of the study

The remainder of this thesis is structured as follows, chapter 2 gives an overview of Bitcoin as an asset class, and also the South African context and backdrop to this study. Following this, the literature that is material to this study is discussed in Chapter 3. Chapter 4 addresses the data used for this study, while Chapter 5 sets out the methodology used. Chapter 6 analyses and discusses the results found, and Chapter 6 concludes the study.

Chapter 2: Overview: Bitcoin & South African financial markets

This chapter sets out an understanding of Bitcoin and the technology behind it. Given that this study tests the effects of including Bitcoin in a South African investment portfolio, it is important to understand how Bitcoin works and the dynamics behind its value and function. In addition, this chapter provides a brief overview of the South African financial markets, which is the context within which a South African investment portfolio would be constructed.

2.1 Bitcoin

Bitcoin is a digital, unregulated and partially anonymous currency which is not backed by any government or legal entity (Grinberg, 2012). Although low barriers to entry have led to a creation of over 1,500 virtual currencies at present, only Bitcoin, as the most established and one of the oldest virtual currencies, has sufficiently credible data with a long enough history available to allow for an analysis of its return and risk factors. Therefore, although coins such as Ethereum have also gained the attention of investors outside of technological enthusiasts, these invariably do not have a long enough data history to satisfy the needs of a statistically robust analysis. Most cryptocurrencies, including Bitcoin, rely on peer-to-peer network and cryptography, through the blockchain system, to maintain its market integrity. These technical aspects are further discussed in the next section.

Historically, when transacting money, people have relied on intermediaries such as banks or governments to ensure the integrity of transactions. Intermediaries are essential for authentication and record keeping, and this also applies to digital transactions. Due to the nature of digital transactions, specifically the fact that they are essentially files, they are easy to reproduce. This creates the double spending problem, which is the act of spending the same unit of value more than once. Until the development of the blockchain concept, this has prevented the evolution of peer-to-peer digital transactions.

Bitcoin first appeared in a 2008 white paper authored by a person using the pseudonym, Satoshi Nakamoto (2008), which detailed the innovation of a peer-to-peer digital cash system called Bitcoin. Bitcoin allows online payments to be transacted without an intermediary, and is based on blockchain technology, which is a decentralised database system that allows it to work as a means of transacting and transferring value. This database keeps records of all digital transactions, in effect replacing the central administrator (such as the bank network) that is

required in traditional transacting networks. When a digital transaction is carried out it is, together with all other transactions that have occurred in the last ten or so minutes, grouped together in a cryptographically protected block and sent out to the entire network of ledgers (Thompson, 2017). Once the peer-to-peer network has validated the block, it is approved, time-stamped and added to the chain in chronological order (Thompson, 2017). Thus, once a transaction has taken place and has been approved it cannot be reversed, unlike electronic fiat transactions, because this would mean undoing all the subsequently validated blocks to reverse the transaction. The chain of blocks shows every transaction made in the history of that blockchain, and is continuously updated so that every ledger in the network is the same. The blockchain database is secure by design. Once recorded, the data in the given block can't be altered retroactively without alteration to all subsequent blocks and ledgers in the network – something that is nearly impossible to do within the very large distributed network of Bitcoin users.

Bitcoin has been dubbed 'the gold of Millennials' (Burniske & White, 2016), a reference to the fact that, like gold, its supply is independent of any one authority and it is divisible into small amounts. Bitcoin relies on so-called 'miners' that use computer resources such as electricity and computing time to solve cryptographic problems. Solving this problem validates the block of transactions (Nakamoto, 2008). Solving the problem first is rewarded in the form of newly issued Bitcoin. Bitcoin runs off a fixed payment scheme that is adjusted every 2016 blocks. This is done to ensure that the finite eventual supply of 21 million Bitcoins are distributed at a steady rate. The number of Bitcoins issued every ten minutes halves every 4 years, with the next halving projected for the end of 2019 (Grinberg, 2012). Current Bitcoin issuance is estimated to be about 12.5 Bitcoins every ten minutes. This deflationary characteristic allows Bitcoin to further mimic gold features. Bitcoin's fixed supply allows it to stay free of any central bank-like intervention. (ECB, 2012).

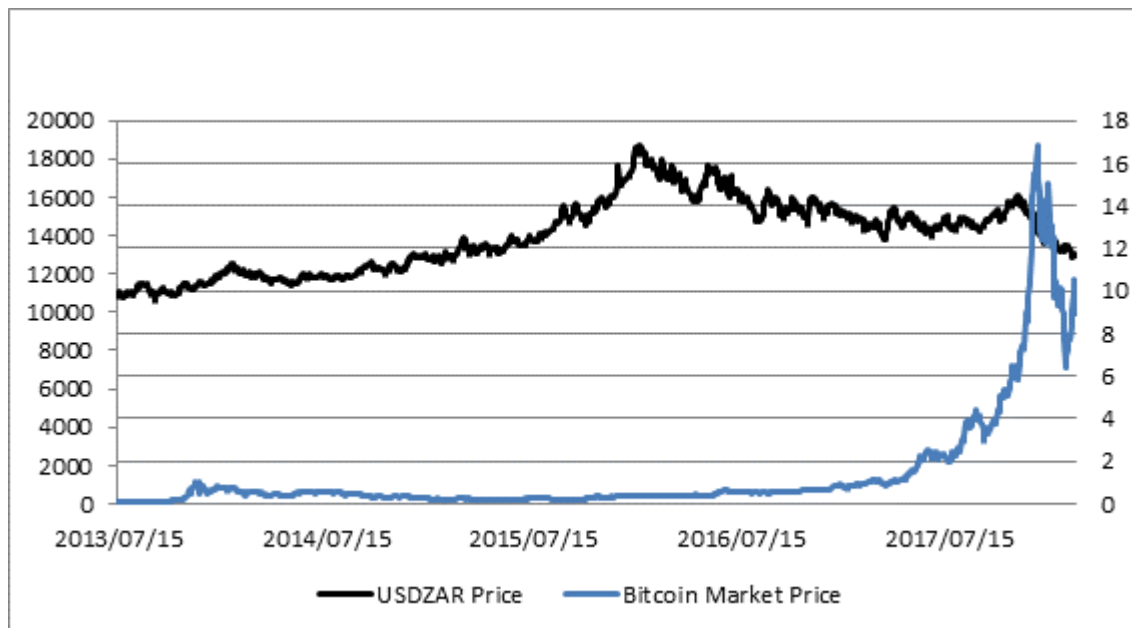
Bitcoin usage has grown tremendously over the last few years, with a market capitalisation just over \$106.45 billion as of July 2018 (CryptoCurrency Market Capitalisations, 2018) (see Appendix A1.1). Bitcoin's price is driven by the scarcity of the coin (supply) and demand for it on open exchanges. Bitcoin has increased its liquidity over the years, further reducing the liquidity risk of the coin. Bitcoin now displays high liquidity, with an average transaction volume of over 109,857 BTC or \$ 678,185,267.44 (subject to change in BTC value) daily as

of July 2018 (see Appendix A1.2) (Bitcoin currency statistics, 2018). Online Bitcoin exchanges are open seven days a week, which allows for trading on an ongoing basis (Michie, 1999).

Bitcoin has low costs and can be used to make quick international payments at a fraction of the cost. A report by the World Bank documented the average remittance fee for sending \$200 dollars, in the second quarter of 2017, as 7.32% on the transaction size (The World Bank, 2017). Bitcoin, has never had an average transaction fee over \$9 per transaction as of September 2017 (Bitinfochart, 2018). However, Bitcoin fees increased to an all-time high in December 2017 (\$55 per transaction) as the amount of transactions caused the block size to be capped (1MB), hence, leading to an increase in fees". These fees have since decreased to as low as \$0.8 in July 2018 (Bitinfochart, 2018). Bitcoin found solutions to this problem by means of a consumer being able to transfer to multiple counterparts with one transaction. Bitcoin, similar to the US Dollar, is not redeemable for any type of money or any commodity (Grinberg, 2012). Bitcoin exchanges stay open on weekends, which allows for more opportunity for investors. Exchanges that are available for trading seven days a week, offer a valuable characteristic for investors to possibly enhance their returns (Michie, 1999).

Bitcoins are typically stored in e-wallets, and like cash in a wallet, they can be stolen. Hackers can steal large sums of Bitcoins off an e-wallet and as such, e-wallets should be backed up and secured. This threat of theft, Bitcoin's perceived value fluctuations, Bitcoin's lack as a store of value and its relatively small market cap (\$106.45 billion as of July 2018, CryptoCurrency Market Capitalisations, 2018) compared to mainstream financial assets, drive the high return volatility in this market (Grinberg, 2012). When comparing Bitcoin's daily volatility to that of gold bullion or even the South African rand (ZAR), Bitcoin's volatility is much higher (see Appendix A1.4 and seen in Figure 1.1 below). This higher volatility may limit Bitcoin's ability to enhance the risk-return profile of a typical South African portfolio, especially considering that Bitcoin's price has plummeted five times between 2011 and 2018, falling more than 50% on all occasions. In 2017 Bitcoin plunged again on the back of the news that U.S Securities and Exchange Commission (SEC) rejected the Winklevoss twins Bitcoin exchange-traded fund (ETF) (Beer, 2017) (see Appendix A1.4). However, compared against other cryptocurrencies such as Litecoin and Ethereum (two of the biggest competitors to Bitcoin in terms of market capitalisation), Bitcoin displays less daily volatility (see Appendix A1.3). Thus, Bitcoin is the least volatile option when testing for the diversification properties of cryptocurrencies within a traditional investment portfolio, including within the South African investment context.

Figure 1.1- Bitcoin USD price versus the USD/rand cross rate



Source: Bloomberg

Figure 1.1 displays the historical daily close market price of Bitcoin denominated in US dollars, compared to the historical daily close market price of the USDZAR exchange rate cross

Figure 1.1 above shows Bitcoin's US dollar-price volatility compared to that of the US dollar/rand exchange rate volatility. This shows that Bitcoin exhibits higher volatility compared to a currency such as the South African Rand, and should be looked at with caution when adding it into a portfolio for diversification enhancement. It must also be stated that once a transaction has taken place and has been approved it cannot be reversed, unlike electronic fiat transactions.

At the base of it, Bitcoin is used to transfer value or hold onto as an investment. An investor can buy and sell Bitcoin on many different exchanges, and then subsequently store Bitcoin in electronic "e-wallets". The first step would be to set up an e-wallet, which comes in several different forms, either as online wallets (as an exchange platform, independent wallet, or mobile wallet) or an offline wallet (hardware devices) (Acheson, 2018). Once a wallet is set up, usually through a cryptocurrency exchange platform, the investor can begin to purchase and sell Bitcoin.

Cryptocurrency exchanges buy and sell the Bitcoin on the investor's behalf, with the largest Bitcoin exchanges in the world being Bitfinex, Coinbase and Bitstamp (Acheson, 2018). The

investor is required to register with an exchange and set up an account with identification verification (typically proof of identity and address). Most exchanges accept payments via bank transfer or credit card. Once the exchange has received the investor's funds, it allows the investor to buy or sell Bitcoin at either, specified order levels or at the current spot level. The exchange buys or sells the Bitcoin on the investor's behalf, and subsequently deposit it into the exchange platforms generated wallet (Acheson, 2018). Once the transaction has been verified and approved, investors can keep the Bitcoin in their wallet on the exchange or choose to transfer it to another wallet of their choice.

In addition, the Chicago Board of Exchange (CBoE) has allowed the trading of Bitcoin futures to be added to its futures platform. This give investors another way to invest in Bitcoin without having to go through cryptocurrency exchanges. The inclusion of Bitcoin to the futures exchange is argued to have enhanced the credibility, transparency and liquidity of Bitcoin, which also led to a more efficient price discovery and a centralised clearing that is cash settled. The futures exchange can be used to trade Bitcoin without mass capital to gain large exposure (leverage), or to hedge current Bitcoin exposure. The futures are based off the Gemini Trust Companies auction price for Bitcoin, which is denominated in US dollars. This adds a new way to invest in or hedge Bitcoin risk for an investor. A South African investor would, however, have to open a CBoE account to trade these futures, which are US dollar denominated.

2.2 South African financial markets and asset classes

The South African bond and equity markets are highly developed by global standards (Hassan, 2013; JSE, 2013). This section will give an overview of South Africa's financial markets, and also highlight some of the key differences between the South African markets and developed markets such as the US. As will be shown in Chapter 3, most of the previous Bitcoin diversification studies were done in the US. However, Bitcoin could have a different effect and result as a portfolio diversifier in an emerging market such as South Africa, compared to in a developed market such as the US.

South Africa's bond market is the largest in Africa by market capitalisation and liquidity, and was the 4th largest bond exchange in 2013 based on value of bonds traded (JSE, 2013). In 2013, South Africa's bond market had roughly R1.8 trillion in debt listed instruments outstanding (JSE, 2013). South Africa's debt market is the most developed debt market in Africa, with over R25 billion traded daily. By the end of 2016, South Africa's bond market turnover, of around

10 times, was far greater than China (0.54) and Japan (1.1). This shows higher liquidity and price discovery compared to other emerging bond markets (Asian Development Bank, 2017; Department of the National Treasury, 2017).

South Africa's equity market, known as the Johannesburg Stock Exchange (JSE), is one of the world's twenty largest stock market and the sixth largest in emerging markets (Hassan, 2013). Emerging market stock exchanges exhibit higher than average returns for higher than average risk, which has a significant effect on the portfolio selection and allocation process (de Groot & Zhou, 2015). The JSE is the largest equity market in Africa with over 400 listed firms and a market capitalisation of over \$900 billion in 2017 (StockMarketClock, 2017). The JSE makes up 6% of emerging market exposure and thus, its value is largely effected by global fund flows (Serhan, 2014). The JSE has huge significance in the national economy and is a highly-capitalised economy. The JSE's market capitalisation to GDP being that of 190% with only Hong Kong (914%) and Singapore (224%) being higher (Hassan, 2013). South Africa's long-run equity premium is around 6%, one of the highest when compared to advanced economies (Hassan & Van Biljon, 2010). Unlike the US market, for example, South Africa's equity market is highly concentrated with over 80% of the market capitalisation coming from the top 40 companies (Courtney Capital, 2017), and this concentration adds risk to market portfolio diversification. Emerging markets such as South Africa have high concentration, higher long-run excess returns with proportionally lower volatility attached, and in turn have different equity Sharpe ratios. These Sharpe ratios differentiate the South Africa market from that of developed markets(de Groot & Zhou, 2015).

With regards to the money market, on a basic interest rate level, the South Africa JIBAR has historically always been much higher than that of the US LIBOR rate (see Appendix A1.6). This differential affects the asset allocation and weighting to cash in a South African portfolio. South African's inflation has also been much higher than that of the US in recent history (see Appendix A1.5). This means that investors in South Africa need to find the means to beat a higher inflation benchmark through greater returns. South Africa's portfolio allocation will be differentiated to that of the US as investors try beat a higher inflation rate.

2.3 Conclusion

Cryptocurrencies, and Bitcoin specifically, are a potentially new asset class that may provide investors with diversification benefits in a portfolio context. Studies on this, to be discussed in

Chapter 3, have all focused on the US. However, as discussed above, South Africa's bond, equity, returns, risk, cash and CPI are potentially different from that of a developed country such as the US. These factors may have a differentiating effect on the efficient frontier of a South African asset portfolio compared to that of a developed market such as the US. South Africa's political risk, inflation targeting framework, currency risk, concentration risk, liquidity risk and excess returns over developed markets all pose as differentiating factors when assessing Bitcoin's diversification ability in a portfolio. Thus, it is possible that the results of studies conducted in developed markets may not hold for an emerging market, or in this case, the South African market.

Chapter 3: Literature review

This chapter examines, firstly, the literature on portfolio diversification, followed by a broad overview of academic studies on cryptocurrencies, especially Bitcoin. Therefore, the empirical evidence on Bitcoin's portfolio diversifying ability, asset allocation effects and the multiple statistical frameworks are reviewed and assessed. Also, included in this chapter is past literature findings on Bitcoin's risk-return characteristics, to better understand this asset as a possible contributor to a diversified portfolio. Lastly, given that a component of this study is a comparison of Bitcoin and gold as portfolio diversification assets, this chapter will discuss past studies of gold as a diversifier and 'safe-haven'. This Section will be broken into five parts, namely, Diversification theory and methods (3.1), Bitcoin as an asset class (3.2), Bitcoin diversification: Empirical evidence and findings (3.3), Gold as a diversification asset: Empirical evidence and findings (3.4) and Bitcoin vs Gold: hedging abilities (3.5).

3.1 Diversification theory and methods

The efficient market hypothesis (EMH) was first introduced by Fama (1970). The efficient market hypothesis states that a market security's price is fully reflective of all available information at a point in time. There are different degrees to the EMH, namely the weak-, semi-strong- and strong forms. In its strong form, the efficient market hypothesis states that an investor cannot obtain above the average risk-adjusted returns based on all available information (past, current, public, and private) (Fama, 1970). The efficient market hypothesis only holds under certain assumptions, which has led to much criticism of this hypothesis, as many of these assumptions do not hold in the real world.

The idea that markets are efficient has led to the pioneering research on capital markets done by Markowitz (1952, 1959) and Tobin (1958). Modern Portfolio Theory, as set out by Markowitz (1952), together with Tobin (1958)'s Separation Theory, underpin the idea that risk-averse investors will allocate assets to maximise their utility, while looking to maximise their returns and minimise their risks.

Markowitz (1952)'s Modern Portfolio Theory sets out to determine a specific investor's optimal portfolio through the best combination of return to risk, by use of a mean-variance optimisation approach. Standard deviation and variance are introduced as the risk measures

used to determine a portfolio's risk (Markowitz, 1952). Markowitz (1952) found that investors prefer higher returns to lower returns, but mainly that investors preferred stability in returns. The efficient frontier is defined as the most mean-variance efficient portfolios (optimal portfolios); thus, the highest expected return for a given level of risk, or lowest expected risk for a given level of return. Tobin (1958)'s Separation Theory builds on Markowitz (1952)'s theory, by accounting for an investor's risk and return preference through a series of indifference curves. These two theories underpin the mean-variance optimisation for portfolio selection.

As mentioned above, these theories are based on assumptions that do not hold true in the real world. The mean-variance assumptions set out by Markowitz (1952) assumes that returns are fully explained by mean and variance, and not by skewness and kurtosis. Thus, the mean-variance optimisation strategy assumes asset returns are normally distributed, and therefore that variance captures upside and downside risk equally. Variance therefore, assumes favourable upside deviation over the mean return, and unfavourable downside deviation from the mean as equal (Fishburn, 1977). This means that investors equally weight positive and negative returns. In reality, asset returns are generally not normally distributed, and investors do not view risk symmetrically; rather, they are more concerned with the downside risk (usually below a specific benchmark) (Leland, 1999).

It has been shown that Bitcoin does not display normal return distributions, and experiences excess kurtosis and positive skewness⁶. Excess kurtosis means that the return distribution displays steeper peaks and fatter tails. It is also known that the JSE displays non-normally distributed returns (Mangani, 2007). Therefore, as the theoretically correct mean-variance optimisation framework does not consider the two moments described above, it is not appropriate for diversification approaches involving assets with non-normally distributed returns.

Mean-semivariance was therefore introduced as a more accurate risk measure, substituting semivariance for variance within the framework. The concept behind semivariance is that investors do not have symmetric preference for over-performance as they have aversion for

⁶ See, for example, Baur, Lee and Hong (2015); Yermack, (2013); Burniske and White, (2016); Wu and Pandey (2014); Brière, Oosterlinck and Szafarz (2015); and Eisl, Gasser and Weinmayer (2015)

underperformance of the portfolio. Investors are therefore considered to be more concerned with the downside risk (Leland, 1999). The downside risk measure only captures risk below a certain threshold or benchmark, and returns above the benchmark (even if negative) are not considered to contribute to risk. According to Markowitz (1959), the semivariance risk measure produces a more reliable and accurate portfolio than using the variance risk measure.

There has been much debate over the complexity of implementing the mean-semivariance optimisation, specifically with regards to the calculation of the co-semivariance matrix. Only fairly recently did Estrada (2007), present a solution to this complex co-semivariance matrix, by determining the periods for which assets jointly underperform (thus the portfolio underperforms) the target return.

This study thus follows Estrada (2007) in practice, and Markowitz (1959) in principle, in its implementation of the mean-semivariance approach.

3.2 Bitcoin as an asset class

Bitcoin is a relatively new digital phenomenon (originating only in 2009), and from an academic perspective, most of the research on Bitcoin involves security, laws, money laundering and the actual computer science behind it. This section lays out some of the recent literature on the factors effecting Bitcoin.

Much of the academic discussion on cryptocurrencies, including Bitcoin, is found in legal journals, specifically with regards to the legality and regulatory issues surrounding Bitcoin. Turpin (2014), for example, come to the conclusion that Bitcoin is not illegal under nearly every county's legal framework, and that governments currently do not have the ability to access the Bitcoin network directly. This is an important consideration, as it underpins the legality and delocalisation of Bitcoin as an investment option. Doguet (2012) concurred with Turpin (2014), and argued that, due to the self-administrating money supply system, Bitcoin eliminates the need for government interventions. Both studies state that money laundering and criminal abuse of Bitcoin cannot be denied. These criminal activities add a risk to the coin's viability as an investment, due to governments' duty to shut down criminal activity.

There is still a lack of academic research into the investment qualities of Bitcoin. Grinberg's (2012) study is one of the first to address Bitcoin from an economic perspective. This study

analyses Bitcoin as a digital currency. Bitcoin displays an advantageous micro-currency feature, making it divisible to the eight-decimal place (Grinberg, 2012). On the negative side, Bitcoin is susceptible to competition a result of low barriers to entry (Grinberg, 2012). This has been seen, with over 1600 cryptocurrencies in existence as of July 2018 (CoinMarketCap, 2018). These low barriers may, however, in one sense may be advantageous to potential investors, as it allows for strong competition and the promotion of lower transaction fees and similar return payoff products. A further risk is government regulations, which could pose a threat to Bitcoin in the future (Grinberg, 2012).

Much debate concerns the nature of Bitcoin and cryptocurrencies in general, namely whether they are a new asset class or a currency. Bitcoin has long been thought of as a means of payment and hence a currency, but its unique characteristics show investment-like tendencies (Baur, Lee & Hong, 2015). Evidence suggests that a large proportion (and possibly the majority) of Bitcoin users hold it for investment purposes (albeit of a speculative nature), rather than for transactions (Baur, Lee & Hong, 2015). Furthermore, Bitcoin's excessive volatility makes it a poor store of value, unit of account, and thus a poor medium of exchange (Yermack, 2013; Burniske & White, 2016; Wu & Pandey, 2014). Thus, due to cryptocurrencies unique features and lack of similarity with traditional currency features this study will consider Bitcoin as a new investment asset rather than a currency.

3.3 Bitcoin diversification: empirical evidence

The academic literature on the diversification properties of Bitcoin within in a portfolio context is scarce. In one example, a study by Baur, Lee and Hong (2015) finds that Bitcoin, when compared against traditional asset classes (sixteen assets in total), exhibits the highest returns and standard deviations. Some of the asset classes included in this study are the S&P 500 (a US equity index), S&P 600 (another US equity index), spot gold, spot silver, the Euro/US Dollar exchange rate, the British Pounds/US Dollar exchange rate, a crude oil index, a natural gas index, the Bloomberg US Corporate Bond Index and the Bloomberg USD High Yield Corporate Bond Index. Furthermore, Bitcoin displays return asymmetry, large negative skewness and high kurtosis compared to the other assets⁷, which indicates that on a relative basis Bitcoin is more susceptible to tail events.

⁷ See Baur, Lee & Hong (2015); Yermack (2013); Burniske and White (2016), and Wu and Pandey (2014)

Similarly, Wu and Pandey (2014) find that Bitcoin exhibited extremely low correlations with all major asset classes tested, namely stocks, bonds, real estate, commodities, currencies and the Chicago Board Options Volatility Index (VIX). These authors take a developed market view. Stocks are represented by the S&P 500 index (SPX), Bonds are represented by the Bloomberg USD Investment Grade Bond Index (BIG), real estate is represented by the TSE NAREIT Total Return index (FNARTR), and commodities are represented by the S&P Spot Commodities Index (SPG-SCI). The Bloomberg Dollar Spot Index (BBDXY) is used as proxy for currency – in this case, the US dollar.

Wu and Pandey (2014), in line with most investors' preferences, observe long-only portfolios. From the perspective of a U.S investor, Bitcoin's returns are largely independent of the asset classes above, and would inherently enhance the portfolio (Wu & Pandey, 2014). Bitcoin is added under different portfolio optimisations to test its diversification effect. The long-only portfolios are optimised through minimising total variance, minimising downside semi-variance and maximising risk adjusted returns under the Sharpe ratio (excess return adjusted for total risk), as well as the Sortino ratio (adjusted for downside risk). Wu and Pandey (2014) discover that, in all cases, Bitcoin increases portfolio returns and reduces the probability of incurring a loss, which suggests that adding Bitcoin under all optimised portfolios enhances the portfolios' efficiencies. An interesting discovery is that, when optimising the portfolio through the Sharpe ratio or Sortino ratio, the weighting of Bitcoin in the portfolio becomes 100%. This is due to the Sharpe and Sortino ratio optimisation being largely influenced by high returns, although the recent decline in Bitcoin's dollar prices may imply that these results may no longer hold as strongly as before.

Although they take a US investor's view, the studies by Brière, Oosterlinck and Szafarz (2015), and Eisl, Gasser and Weinmayer (2015) also include emerging market indexes for bonds and stock markets as portfolio assets. These studies test the inclusion of Bitcoin into an already well-diversified portfolio. Brière et al. (2015) use the traditional mean-variance framework and methodology as set out by Markowitz (1952) to assess Bitcoin as a portfolio diversifier. A major criticism of using the Markowitz (1952) mean-variance framework for this purpose is that Bitcoin's returns are not normally distributed (Eisl, Gasser & Weinmayer, 2015). Eisl et al. (2015) used the Mean-CVaR framework that was developed by Rockafella and Uryasev (2000), which suggests for no normality, to assess Bitcoin's portfolio diversification characteristics – also in an already well-diversified portfolio.

Brière et al. (2015) use weekly Bitcoin data taken from Bitcoincharts over the investment period 23 July 2010 to 27 December 2013 (41 months), whilst Eisl et al. (2015) use monthly data taken from Coindesk over a 57-month period (July 2010- April 2015). It is worth noting that Eisl et al. (2015) use the MSCI Emerging Market Index and the Bloomberg Emerging Market Sovereign Bond Index as proxies for emerging market equities and bonds respectively. South Africa's weighting is less than 6% in the MSCI (MSCI, 2017), and a mere 2.1559% (the 14th biggest component) in the Bond Index (Bloomberg terminal, 2018).

Although both studies, in accordance with the previously discussed literature, find extremely low correlations between Bitcoin and all asset classes tested (see Tables 3.1 and 3.2), these correlations must be interpreted with caution in a bull trend (Brière et al. 2015). Correlations are known to be unstable and can change a vast amount under stress. Therefore, given that the data analysed by Brière et al. (2015) only captured Bitcoin in a bullish trend (July 2010 – December 2013), the correlation coefficients may change significantly over time, especially in a bear trend.

Noteworthy is also that this study shows that Bitcoin's correlations with emerging market equities (Stocks emg) and bonds (Gvt bonds emg) are lower than with their developed market equivalents (Stocks dvp and Gvt bonds dvp) (see Table 3.1, below).

Table 3.1 - Briere, Oosterlinck, and Szafarz (2015) correlation matrix results.

Table 2: Correlations (in %)

	BTCs	Euro	Yen	Stocks dvp	Stocks emg	Gvt bonds dvp	Gvt bonds emg	IL bonds wld	Corpo bonds wld	Gold	Oil	Real estate	Hedge funds
BTCs	—	—	—	—	—	—	—	—	—	—	—	—	—
Euro	-4	—	—	—	—	—	—	—	—	—	—	—	—
Yen	-6	21***	—	—	—	—	—	—	—	—	—	—	—
Stocks dvp	5	-53***	4	—	—	—	—	—	—	—	—	—	—
Stocks emg	4	-45***	6	80***	—	—	—	—	—	—	—	—	—
Gvt bonds dvp	8	-64***	-74***	16**	20***	—	—	—	—	—	—	—	—
Gvt bonds emg	3	-27***	-5	34***	53***	39***	—	—	—	—	—	—	—
IL bonds wld	14*	-60***	-44***	23***	30***	84***	48***	—	—	—	—	—	—
Corpo bonds wld	10	-70***	-38***	38***	49***	81***	61***	81***	—	—	—	—	—
Gold	14*	-38***	-36***	21***	31***	49***	31***	50***	48***	—	—	—	—
Oil	-1	-34***	-6	50***	47***	14*	20***	23***	21***	30***	—	—	—
Real estate	0	13*	15**	63***	66***	-11	46***	2	14*	0	28***	—	—
Hedge funds	9	-34***	15**	77***	71***	7	32***	23***	37***	17**	49***	60***	—

This table displays the correlation matrix between the weekly returns of the 13 assets under study (USD)

The Eisl et al. (2015) study reconfirms the extremely low emerging market correlations seen in the study by Brèire et al. (2015). Table 3.2 shows that the MSCI Emerging Market Index (emerging market equity) and the Bloomberg Emerging Market Sovereign Bond Index (emerging market bonds) have the lowest correlations out of the fixed income and equity indexes. This enhances the idea that Bitcoin could be a more powerful diversifier in an emerging market such as South Africa, and alter the portfolio risk-return profile.

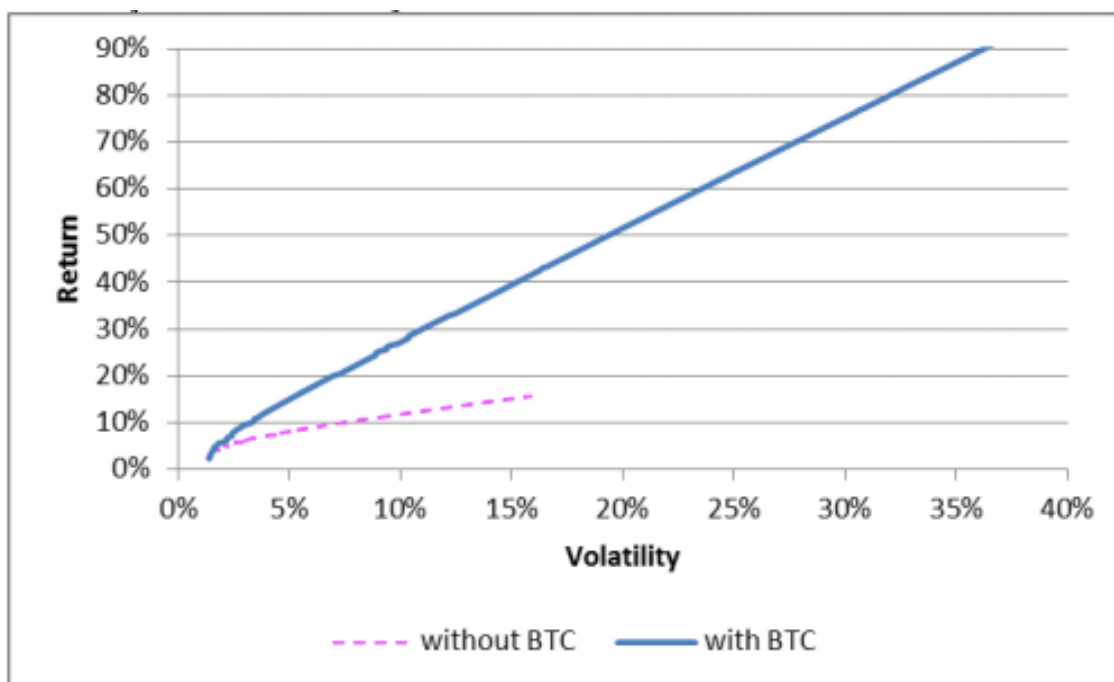
Table 3.2 – Eisl, Gasser and Weinmayer (2015). Correlation Matrix.

	Correlation Matrix - Coefficients											
	coindeskBPI	bcor	bgs	bhyc	blc	engl	hfrxgl	msciem	mscifm	msciworld	mxea	spwici
bcor	0.13											
bgs	0.09	0.83										
bhyc	0.15	0.80	0.43									
blc	0.00	0.79	0.59	0.70								
engl	-0.03	0.40	0.22	0.35	0.46							
hfrxgl	0.15	0.47	0.15	0.74	0.50	0.32						
msciem	0.03	0.68	0.36	0.83	0.82	0.39	0.69					
mscifm	0.12	0.39	0.15	0.60	0.43	-0.02	0.65	0.60				
msciworld	0.16	0.62	0.25	0.87	0.67	0.32	0.83	0.84	0.65			
mxea	0.14	0.68	0.33	0.88	0.72	0.33	0.80	0.85	0.65	0.97		
spwici	0.09	0.41	0.25	0.60	0.40	-0.12	0.55	0.58	0.50	0.61	0.60	
tipus	0.06	0.63	0.70	0.30	0.47	0.46	0.01	0.24	-0.01	0.04	0.12	0.02

This table represents the pairwise correlation coefficients of the assets included in the portfolio under study

In addition, Brèire et al. (2015) constructed a mean-variance efficient frontier to graphically display the effect of Bitcoin on the portfolio optimisation. Figure 3.1 shows the effect Bitcoin's inclusion has on the mean-variance efficient frontier. Bitcoin can be seen to enhance the efficient frontier under the mean-variance framework, by steepening the frontier drastically. In this study, due to Bitcoin's extremely volatile price nature, it was too volatile to be included in the minimum-risk portfolio. However, once the risk tolerance of an investor is slightly increased, the mean return for a given level of risk increases sharply, thus improving the risk-return trade off.

Figure 3.1 - Briere, Oosterlinck and Szafarz (2015) mean-variance efficient frontier



This figure displays two efficient frontiers set out by Brière et al. (2015). The ‘without BTC’ frontier is derived from the 13 assets under study less Bitcoin, and the ‘with BTC’ frontier is derived from the same assets, but includes Bitcoin.

In an equally-weighted portfolio, Bitcoin drastically improves the Sharpe ratio according to the two abovementioned studies. However, a 7.7% portfolio weighting in Bitcoin (*i.e.* as an equally-weighted portfolio of 13 assets including Bitcoin) would not be acceptable for a reasonably risk-adverse investor, as its inclusion more than doubles the volatility of the portfolio in the Brière et al. (2015) study.

Eisl, Gasser and Weinmayer (2015) further this work by looking at the effect of Bitcoin under multiple portfolio frameworks. As with the previous studies, they take the perspective of a US investors, and use Conditional Value-At-Risk (CVaR) as a risk measure. Eisl et al. (2015), use backtesting techniques to derive out-of-sample returns based on optimal portfolio weights and risk-return ratios to derive optimal weights under four different portfolio frameworks, namely unconstrained, -100%/+100% long-short, long-only, and equally weighted. Consistent with

Brière et al. (2015), Eisl et al (2015) found that Bitcoin's weighting was the highest in the equally weighted portfolio, and that the risk (CVaR) nearly doubles. The study further finds that, across the board, the weighting of Bitcoin is relatively low, regardless of the portfolio framework. Under all optimal portfolios, Bitcoin increases the efficiency of the risk-return ratio in this study. The unconstrained framework yielded the second highest weighting, with the biggest improvement in the risk-return ratio (see Table 3.3).

Table 3.3 – Eisl et al. (2015). Detailed results of each portfolio optimisation framework.

Portfolio Optimization Framework	Mean Monthly BTC Weight	Mean Monthly Return	Mean Monthly CVaR	Mean Monthly Risk-Return Ratio
Equally-Weighted BTC	7.69%	1.93%	1.01%	3.75%
Equally-Weighted No BTC	-	0.38%	0.64%	2.32%
Long-Only BTC	2.09%	0.43%	0.60%	1.04%
Long-Only No BTC	-	0.28%	0.51%	0.60%
Unconstrained BTC	6.65%	5.21%	1.94%	5.88%
Unconstrained No BTC	-	0.43%	0.34%	1.81%
-100%/+100% BTC	1.65%	1.03%	0.53%	2.77%
-100%/+100% No BTC	-	0.43%	0.34%	1.81%

This table represented the mean return, CVaR and risk-return ratio of each portfolio. Data is presented as monthly means.

In summary, using different frameworks help to assess Bitcoin's diversification properties under different methodologies. Eisl et al (2015) found similar results to that of Brière et al. (2015) and Wu and Pandey (2014), and in this case the mean-variance and mean-CVaR frameworks therefore seem to produce similar results.

Finally, a different methodology to the mean-CVaR and mean-variance methodologies has been used to test Bitcoin's diversification ability (Carpenter, 2016). Again, in the context of a US investor, Carpenter (2016)'s study has no emerging market proxy component. The methodology used is a modified version of the mean-variance framework used by Breire et al. (2015), with an additional 'return penalty' added to account for Bitcoin's high kurtosis and skew (Carpenter, 2016). Thus, historical returns with a magnitude-reducing 'return penalty' are used to calculate expected returns. Using this framework, the findings of Brère et al. (2015) and Eisl et al. (2015) are again confirmed by using backtesting techniques.

However, the volatility of Bitcoin is a major drawback when it comes to adding it to a portfolio. Some studies conducted from a US perspective find that Bitcoin exhibits low (and sometimes

negative) correlations with most of the currencies investigated⁸. Yermack (2013)'s investigation into the characteristics of Bitcoin over the period July 2010 to March 2014, however, confirms that its daily value is far more volatile than that of any traditional currency. Bitcoin also exhibits close to zero correlation with all widely-used currencies, as well as with gold, making it a potentially strong candidate for diversification purposes.

Table 3.4 - Yermack's correlation matrix results

Correlation Matrix						
	EUR	JPY	CHF	GBP	Gold	Bitcoin
EUR	1,00	0,18	0,61	0,64	0,20	-0,05
JPY		1,00	0,33	0,20	0,07	0,01
CHF			1,00	0,42	0,19	-0,04
GBP				1,00	0,21	-0,02
Gold					1,00	-0,06
Bitcoin						1,00

Table 3.4 shows the correlation between the daily percentage changes in the price Bitcoin, selected exchange rates and gold.

As discussed, Bitcoin's correlation to major currencies and the gold price are near zero, while the major currencies are clearly correlated to each other to a reasonable degree. Yet, given Bitcoin's volatile nature it would, if anything, closely resemble an emerging market currency (Burniske & White, 2016). However, two studies (Burniske & White, 2016; Wu & Pandey, 2014), reveal that Bitcoin also has very low correlations with emerging market currencies. Both studies argue that, due to its extremely low correlations with all asset classes, Bitcoin could be used as an investment asset to further diversify and enhance portfolio efficiencies. These studies therefore reiterate the findings of Yermack (2013) and Baur, Hong and Lee (2015) discussed above. Nearly all previous results thus show that, under all portfolios, Bitcoin improves the risk-return profile and enhances the efficient frontier, at least within the US context.

Bitcoin's low correlations with emerging market currencies raises the possibility that it may have a particularly high diversification benefit in an emerging market context such as South

⁸ For some studies that came to this conclusion, see Baur, Lee and Hong (2015); Yermack (2013); Burniske and White (2016); Wu and Pandey (2014); Brière, Oosterlinck and Szafarz (2015), and Eisl, Gasser and Weinmayer (2015)

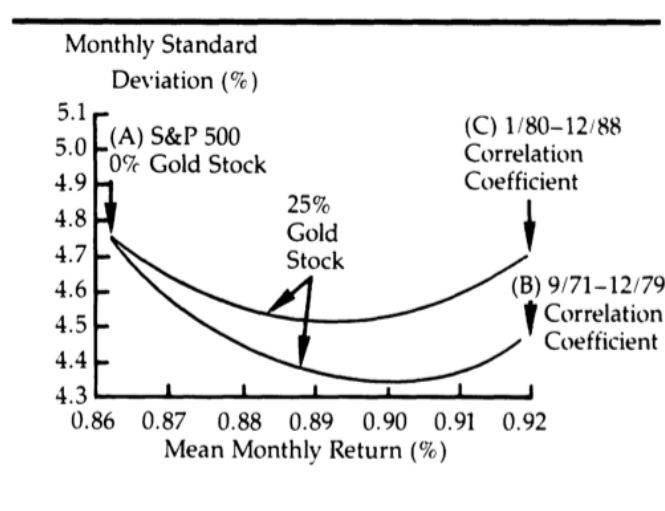
Africa,⁹ a possibility that has to the author's knowledge not been researched before. In addition, in all of the abovementioned studies, the dollar Bitcoin price displayed extremely low correlations with the dollar gold price. It is thus possible that Bitcoin could be a new portfolio diversification tool for South African investors, either substituting or complimenting gold for this purpose.

3.4 Gold as a diversification asset: empirical evidence and findings

It has been well documented that gold can reduce risk in a common stock portfolio (Jaffe, 1989). Thus, Chua, Sick and Woodward (1990) set out to test whether gold stocks or gold bullion was a good diversification option for a common stock portfolio over the period 1971-1988, by firstly determining whether gold is a diversifier that can be used to reduce risk in a common stock portfolio, and secondly, whether gold stocks or gold bullion have the same diversification abilities over the short-term as they have over the long-term. Thus, a basic regression model with monthly data is used to test whether gold bullion and gold stocks held their diversification abilities over the short term (defined as 8-year periods). In this study 25% gold bullion and gold stock holdings is added to a well-diversified portfolio at two separate short-term time periods to see if gold holds its long-term diversification abilities in the short-term. As can be seen from Figure 3.2, Chua et al. (1990) found that gold bullion (stated gold stock on the graph) increased the risk/return profile of the portfolio in both short-term periods, and thus was a useful diversifier in the short-term over the period tested for. Unlike gold bullion, the 25% addition of actual gold stocks decreased the risk/return over the period 1980 – 1988, and only slightly decreased it for the period 1971-1979. Thus, it was concluded that gold stocks do not hold their diversification abilities during the short-term (Chua, Sick & Woodward, 1990).

⁹ See Tables 3.1 and 3.2, which show selected results of the studies of Brière, Oosterlinck and Szafarz (2015) and Eisl, Gasser & Weinmayer (2015), respectively

Figure 3.2 – Chua et al's common stock portfolio with gold bullion.



Source: (Chua, Sick & Woodward, 1990)

This figure shows the addition of 25% Gold Bullion to a common portfolio over two separate time periods, 1971-1979 and 1980-1988.

In a separate study, Jaffe (1989) shows that the correlations between gold bullion and gold stocks to the S&P 500 were low, at around 0.054 and 0.304, respectively. This indicates that gold bullion and gold stocks could both be used to diversify a common stock portfolio without diminishing the average return. Jaffe (1989) further found that gold stocks would reduce systematic risk by less than gold bullion, but that gold stocks would still reduce the overall systematic risk of the portfolio (Jaffe, 1989). Jaffe (1989) thus concluded that both gold bullion and gold stocks display diversification benefits for a common stock portfolio over the long-term.

Gold has been known to display so-called 'safe-haven'¹⁰ attributes, but its 'safe-haven' ability has been questioned under an emerging market view. Studies done by Baur and Lucey (2010) and Baur and McDermott (2010) discuss gold and its ability to be a 'safe-haven'. Baur and McDermott (2010) use GARCH modeling to determine whether gold is in fact a hedge and a 'safe-haven' in different market environments. The study is run over the period 1979-2009, and looks at the hedging and 'safe-haven' abilities of Gold under different return periods (daily, weekly, monthly) (Baur & McDermott, 2010).

¹⁰ Where a safe have asset is an asset that retains (or increases) in value in time of market uncertainty or turbulence

Baur and McDermott (2010)'s findings (Table 3.5) suggest that gold is a 'safe-haven' investment in most developed countries' stock markets, with the daily data displaying the strongest 'safe-haven' signals. This suggests that investors panic buy gold under short-lived crisis moments (Baur & McDermott, 2010). The table below shows gold to be a strong hedge under extreme market conditions (0.01), yet gold was also found to be the only weak 'safe-haven' in emerging markets, with a positive value (0.108) compared to that of developed market's negative values. Overall the conclusion is that gold is gravitated towards in times of crisis, and is therefore seen as a 'safe-haven'.

Table 3.5 - Baur and McDermott (2010) GRACH model daily return results

	Daily Frequency			
	Hedge	0.1	0.05	0.01
<i>Indicies (in USD)</i>				
Emerging markets	0.057***	0.121	0.034**	0.108**
EMU	0.081***	0.142***	0.122	(0.087)***
EU	0.088***	0.138	0.160	(0.088)***
Latin America	0.011	0.020	0.065	(0.028)***
North America	(0.064)***	0.005**	(0.002)	(0.070)***
World	0.062***	0.126*	0.081	(0.015)***

This table represents the results of gold as a hedge or 'safe-haven' asset for daily returns. Negative coefficients in the hedge column indicate a hedge against stocks. Negative or zero coefficients in the extreme market conditions (0.10, 0.05, 0.01) indicate a (strong) or weak safe-haven respectively.

Baur and Lucey (2010) reiterated the findings above, namely that gold is in fact a 'safe-haven' asset. However, these authors found that this applied with respect with stocks only, and not so with regards to bonds. They did, however, find that gold only represents a 'safe-haven' for the short-term (15 trading days), and not so much for the long-term, in accordance with the findings of Chua, Sick and Woodward (1990).

3.5 Bitcoin vs. gold: hedging abilities

A study done by Dyhrberg (2016) examines Bitcoin's hedging abilities using GARCH models for the period 2010 to 2015 and found that Bitcoin was significantly affected by its demand, and less so by shocks in the market. This is a similar characteristic to gold. This seemed to reiterate findings that the gold price is more affected by demand for jewelry than market shocks (Hammoudeh & Yuan, 2008). Gold and Bitcoin have similarities in what factors affect their

price movements the most. It is found that Bitcoin is, like any medium of exchange, affected by Federal fund rate changes (Dyhrberg, 2016). As the fed rate increases, so the dollar appreciates and imports increase. Many trades are done online, using Bitcoin as payment option, and thus the demand for Bitcoin increases. Dyhrberg (2016) found that, similar to gold, Bitcoin reacts to bad news positively, which indicates that it embodies the “safe haven” characteristics”. Finally, these authors conclude that Bitcoin could combine some of the advantages of commodities and currencies, and can be used for risk aversion in a similar way to gold (Dyhrberg, 2016).

These studies above, show through findings, that Bitcoin could have hedging and safe-haven abilities¹¹. Findings above suggest that Gold Bullion displays poor ‘safe-haven’ abilities in emerging markets (Baur & McDermott, 2010) and that the diversification abilities were more prominent over the short term. This gap in literature allows for this study to look at Bitcoin as a possible diversifier and see if it will diversify a South African well-diversified portfolio better than gold over the short term (seven-and-a-half years)

3.6 Conclusion

The literature indicates that Bitcoin’s correlation with emerging market bonds and equities are the lowest of the asset classes tested, and hence its diversification ability may be greater in an emerging market context. The studies further find low correlations between gold and Bitcoin, thus creating a gap in the literature for an investigation of the comparative portfolio effects between the two asset classes within an emerging market. The two key previous diversification studies, namely those of Eisl et al. (2015) and Brière et al (2015), used Bitcoin data over only a three-year and four-year time horizon, respectively, of which both only included a bull trend. By increasing the sample period, a more reliable result may be possible, especially as Bitcoin has recently displayed more bearish behaviour (see Appendix A1.4).

¹¹ Baur and Lucey (2010), Baur and McDermott (2010) and Dyhrberg (2016)

Chapter 4: Data

This chapter discusses the data used in this study, which takes the view of a South African investor holding a well-diversified portfolio comprising of six traditional assets (local and international equity and bonds, South African cash/money market investments, and South African listed property), and then assesses the effect of including Bitcoin and/or gold bullion in this portfolio. The table below shows the asset classes under study. In all cases, monthly total return data was used.

Table 4.1 – Data and sources of data used in this study

Proxy needed	Proxy used	Period	Source
Bitcoin Index	Bitcoin/USD Cross	Aug 2010- Feb 2018	Bloomberg
SA Bonds (ALBITR)	JSE All Bond Index Total Return (ALBITR)	Aug 2010- Feb 2018	Bloomberg
SA All share (JALSHTR)	FTSE/JSE All Share Total Return Index (JALSHTR)	Aug 2010- Feb 2018	Bloomberg
SA Property	FTSE/JSE SA Listed Property Index (JSAPY)	Aug 2010- Feb 2018	Bloomberg
SA Cash/Money Market	SA Short Term Fixed Interest Rate Composite Index (STeFI)	Aug 2010- Feb 2018	Bloomberg
Foreign Bonds	Bloomberg Glob. Dev. Sovereign Bond Index (5-7 year)	Aug 2010- Feb 2018	Bloomberg
Foreign Equity	MSCI World Index	Aug 2010- Feb 2018	Bloomberg
Gold Bullion	Spot Gold/USD Cross	Aug 2010- Feb 2018	Bloomberg
USDZAR echange rate	USDZAR Exchange rate cross	Aug 2010- Feb 2019	Bloomberg

This table shows each proxy used, the period of data collected, and the source of the proxy.

Bitcoin Price Index

The Bitcoin US Dollar cross was taken from Bloomberg and represents the US Dollar value of Bitcoin as quoted by Bloomberg.

JSE All Bond Index Total Return (ABLITR)

The ALBI index is a composite index containing the top 20 South African vanilla sovereign and non-sovereign bonds. Only conventional listed vanilla bonds with fixed rate and fixed maturities are included. These bonds are ranked dually by liquidity and market capitalisation, and bonds with a maturity of less than 1 year are excluded (JSE, 2013). The ABLITR is the total return index, which provides an accurate measure of the South African bond market movements and is thus an acceptable proxy for this study.

FTSE/JSE All Share Total Return Index (JALSHR)

The FTSE/JSE All Share Index is a market capitalisation-weighted index that represents 99% of the full market capitalisation of all eligible equities listed on the JSE main board (JSE, 2013). The FTSE/JSE Africa Index Series was the result of a joint venture between the JSE Limited (JSE) and the FTSE Group (FTSE). The FTSE/JSE All Share Total Return Index is part of this series that took over from the JSE Actuaries indices in June 2002. This index is a total return index with includes dividend income and forms a proxy for South African equity in this study.

South African Listed Property Index (JSAPY)

The South African Listed Property Index is a free float market capitalisation weighted index of the top 20 liquid companies in the Real Estate Investment sector, Real Estate Investment Trusts Sector and, Services Sector (JSE, 2013). All companies in this index must have a primary listing on the JSE. This is the proxy chosen to represent the South African listed property market in this study.

SA Short Term Fixed Interest Rate Composite Index (STeFI)

The Alexander Forbes Short Term Fixed Interest (STeFI) Index was used in this study to approximate the performance of money market instruments in the market. Instruments such as call deposits and Negotiable Certificated of Deposits (NCD'S) represent common liquid instruments, and provide a good proxy of short term markets. The base date for this index is 1 October 2000.

Bloomberg Global Developed Sovereign Bond Index (5 - 7 year)

The Bloomberg 5 to 7-year Global Developed Sovereign Bond Index forms part of Bloomberg's global developed market sovereign bond index series. The Global Developed Sovereign Bond Index (5 to 7 year) includes all developed market fixed interest securities with a remaining term to maturity of between 5-7 years. The Bloomberg Global Developed Sovereign Bond Index is a rule-based, market-value weighted index devised to measure the local fixed-rate currency, and public obligations of developed countries. The base currency is the US dollar, and the index contains issues from the U.S., Europe, Canada and the Pacific Rim countries. The 5 to 7-year index was chosen to better represent the time horizon for the Bitcoin data in question, thus matching the investment period. This represents the proxy chosen for the foreign fixed income market in this study.

MSCI World Index

The MSCI World Index is a free-float weighted listed equity index, with a base value of 100 as of December 1969. The index captures large and mid-cap representation across 23 developed countries, and captures approximately 85% of the free float adjusted market capitalisation in each country. It does not include emerging markets. For this reason, it has been chosen as the proxy to represent the foreign equity market in this study. (MSCI, 2018)

Gold Bullion

In this study the gold spot price, quoted in USD per troy ounce, was used as the basis for calculating the returns of gold bullion.

This study covers the period August 2010 to February 2018 (*i.e.*, 91 months). The study uses monthly closing data so as to eliminate the noise attached to daily and weekly data. The data that is denominated in Dollars, namely that for Bitcoin, the Bloomberg Global Development Sovereign Bond Index (foreign bonds), the MSCI World Index (foreign equity), and gold bullion, were converted into South African Rand by using the monthly close of the USDZAR currency cross exchange rate.

The following chapter, Chapter 5, will discuss the methodology used for this study.

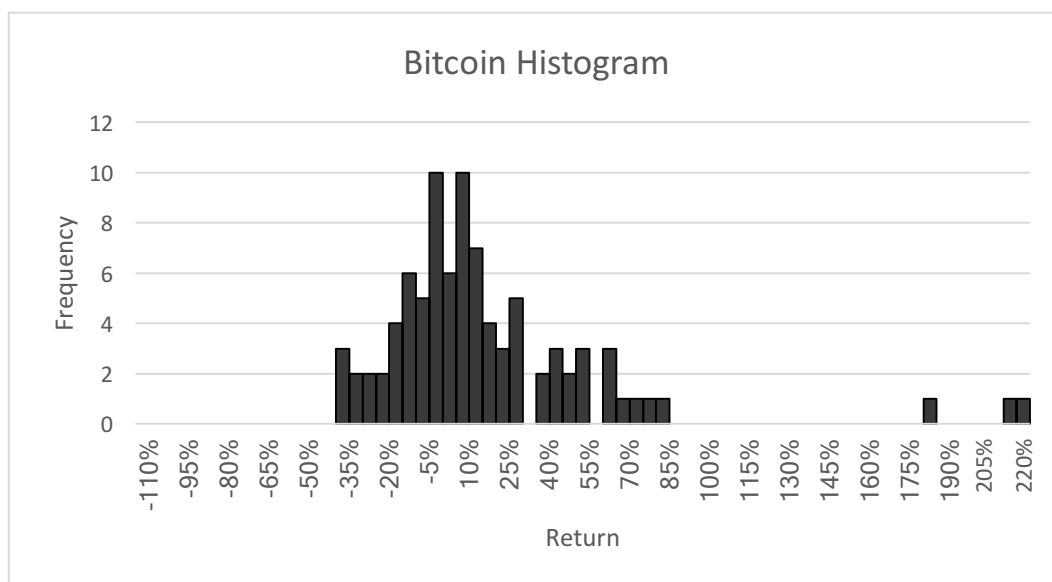
Chapter 5: Methodology

This chapter describes the methodology used in this study, starting with an explanation of the mean-semivariance framework that was used.

5.1 The Mean-Semivariance Framework

The standardised mean-variance framework requires that returns are normally distributed in order to use variance as a risk measure. Bitcoin displays non-normally distributed returns, which calls into question the use of a mean-variance framework as set out by Markowitz (1952) and used by Brière et al (2015). Thus, instead of using the mean variance framework as set out by Markowitz (1952), this study used the mean semivariance framework as set out by Markowitz (1959) and as adjusted by Estrada (2007).

Figure 5.1: Bitcoin Histogram



This figure displays the mean monthly Bitcoin return histogram over the investment period.

Figure 5.1 shows Bitcoin's return distribution as a histogram, which displays a high peak and positive skew. For this reason, Bitcoin does not fit into a normal Capital Asset Pricing (Model CAPM) mean-variance model, which under these conditions would develop an uninteruptable Sharpe ratio (Eisl et al., 2015). The mean-variance framework is likely to under-estimate risk when returns are not normally distributed, as in this case (McNeil et al., 2005).

In the recent study done by Eisl, Gasser and Weinmayer (2015), a Mean-CVaR framework, as set out by Rockafella and Uryasev (2000), was used to assess Bitcoin's portfolio diversification characteristics within an already well-diversified portfolio. The Mean-CVaR framework uses Conditional Value-at-Risk (CVaR) as a risk measure. This is a very complex methodology to implement, in order to be able to evaluate Bitcoin as a possible portfolio diversifying and enhancing asset.

Markowitz (1952)'s mean variance framework has the underlying assumptions that: returns are normally distributed, variance captures both the upside and downside risk and that investors weigh positive and negative returns equally. Variance assumes that the favourable upside deviation over the mean return and the unfavorable downside deviation from the mean are equal (Fishburn, 1977). However, in reality investors do not view risk symmetrically, and are rather more concerned with the downside risk, as measured relative to a specific benchmark. (Leland, 1999).

From the above histogram (Figure 5.1) and in the descriptive statistics described in Chapter 6 (Table 6.1.1), it can be seen that the returns of Bitcoin are not normally distributed, with excess kurtosis and extreme positive skew. Due to Bitcoin's non-normally distributed returns, constructing optimal portfolios and efficient frontiers on an inappropriate risk measure (variance) would lead to misleading results. This is because the mean-variance optimisation does not consider the two statistical moments of skewness and kurtosis, but instead it assumes that returns are fully explained by the mean and variance.

Since this problem arose, many downside risk measures have been developed and used, of which one is semivariance. Downside risk measures only capture risk below a certain threshold or benchmark, and returns above are not considered to be 'risk'. Markowitz (1959) stated that the semivariance risk measure produced a more reliable and accurate portfolio than under the variance risk measure. However, the mean-variance has always been more popular due to its computational ease. It was not until Estrada (2007) presented a solution to the mean-semivariance approach's very complex co-semivariance matrix, that mean-semivariance has become a realistic alternative. The mean-semivariance framework allows for the interpretation of Bitcoin returns by using a risk measure that reflects the downside risk below a benchmark. This more accurately encompasses an investor's needs, and allows for better results when dealing with non-normal distributions.

Mean-semivariance optimisation

Understanding the basics of mean-variance framework is critical to the understanding of the mean-semivariance framework as applied in this study. Markowitz (1952) set out the two equations below under mean-variance optimisation. These equations were used to optimise portfolios and form efficient frontiers.

$$E(R_p) = \sum_{i=1}^n W_i R_i \quad (1)$$

Equation (1) relates to an asset's return and the expected portfolio returns, $E(R_p)$.

Where

$E(R_p)$ is the expected return of the portfolio,

W_i is the weighting of asset i in the portfolio

R_i is the return of asset i

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \quad (2)$$

Equation 2 relates to the variance of a portfolio,

Where

σ_p^2 is the variance of the portfolio,

W_i and W_j are the weightings of assets i and j ,

σ_{ij} is the covariance between assets i and j .

The variance of an asset's returns (σ_i^2) and the covariance between two assets i and j (σ_{ij}), set out by Markowitz (1952), are given by Equations 3 and 4, below.

$$\sigma_i^2 = E[(R_i - \mu_i)^2] = \left(\frac{1}{T}\right) \sum_{t=1}^T (R_{it} - \mu_i)^2 \quad (3)$$

Where

σ_i^2 is the variance of an asset i returns,

R_{it} is asset i 's return in period t ,

μ_i is the mean return of asset i ,

T is the number of observations.

$$\sigma_{ij} = \left(\frac{1}{T}\right) \sum_{t=1}^T (R_{it} - \mu_i)(R_{jt} - \mu_j) \quad (4)$$

Where

σ_{ij} is the covariance between two assets i and j ,

R_{it}, R_{jt} are assets i and j 's return in period t ,

μ_i, μ_j are the mean returns of assets i and j ,

T is the number of observations.

However, Markowitz (1959) also set out the following equation to determine the semivariance and the co-semivariance within a portfolio:

$$\psi_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \psi_{ij} \quad (5)$$

Where

ψ_p^2 is the semivariance of the portfolio,

w_i and w_j are the weightings of assets i and j ,

ψ_{ij} is the co-semivariance between assets i and j .

$$\psi_{ij} = \left(\frac{1}{n}\right) \sum_{k=1}^K (R_{ik} - B)(R_{jk} - B) \quad (6)$$

Where

ψ_{ij} is the co-semivariance between two assets i and j

R_{it} & R_{jt} are the assets i and j 's returns when these returns are less than the benchmark in period k ,

B is the benchmark/target return,

K is the number of times a portfolio underperforms the benchmark.

Much more recent work done by Estrada (2007) suggests a different method for calculating a symmetric co-semivariance matrix. Estrada (2007) sets out the following semivariance equation for an asset i 's return with respect to a benchmark B (ψ_{iB}^2):

$$\psi_{iB}^2 = \left(\frac{1}{K}\right) \sum_{k=1}^K [\text{Min}(R_{ik} - B, 0)]^2 \quad (7)$$

Where

ψ_{iB}^2 is the mean asset semivariance over the investment period,

B is the benchmark/target return,

R_{ik} is the assets (i) returns in period k ,

K is the number of times asset i underperforms the benchmark.

The square root of Equation 7 gives the semideviation of the asset. From the above equation, an equation can be derived to calculate the mean semivariance of a portfolio, which is less complicated than optimising through a co-semivariance matrix process. Instead of calculating a co-semivariance matrix through Equation 6 set out by Markowitz as above, Equation 1 is adopted for expected return, and Equation 7 is adapted to calculate portfolio semivariance, hence leading to Equation 8.

$$\psi_{pB}^2 = \left(\frac{1}{n}\right) \sum_{m=1}^n [\text{Min}(R_{pm} - B, 0)]^2 \quad (8)$$

where R_{pm} is:

$$R_{pm} = \sum_{i=1}^n W_i R_{pi} \quad (9)$$

The elements included in these equations is further discussed in Section 5.2.

5.2 Investment and optimisation process

The steps described below were used in this study to evaluate Bitcoin as a portfolio diversifier and enhancer within an already fully diversified South African portfolio (which includes gold bullion), under different portfolio frameworks.

Step 1

Monthly price data was collected for all the assets classes that were included in the various portfolios, and translated into monthly returns over the investment period using Equation 10 below.

$$R_{im} = \frac{p_m - p_{m-1}}{p_{m-1}} \quad (10)$$

Where

R_{im} is the monthly percentage return of the asset in the m^{th} month

p_m is the month-end price of the asset at the m^{th} month-end

p_{m-1} is the month-end price of the asset one month prior to the m^{th} month-end

Assets that have US dollar denominated prices were converted into South African Rands at the relevant date's (month-end) exchange rates.

$$R_{im} = \frac{(p_m * USDZAR_m) - (p_{m-1} * USDZAR_{m-1})}{(p_{m-1} * USDZAR_{m-1})} \quad (11)$$

The asset classes' monthly percentage returns were calculated using Equations 10 and 11 above.

Step 2

Monthly portfolio returns were calculated by making use of Equation 9 above, and each asset class' monthly percentage returns were multiplied by a randomly simulated weight assigned to it. From this a monthly portfolio return was generated. Monthly portfolio returns were calculated by expanding Equation 9 as follows:

$$R_{pm} = \sum_{i=1}^n W_i R_{im} \quad (9)$$

$$\text{Expanded : } R_{pm} = W_1 R_{1m} + W_2 R_{2m} + \dots + W_8 R_{8m}$$

Where

R_{pm} is the monthly portfolio return for month m

$W_1, W_2 \dots W_8$ are randomly simulated weights assigned to the individual assets over the investment period.

$R_{1m}, R_{2m} \dots R_{8m}$ are monthly returns of the individual assets over the m^{th} month

n is the number of asset classes in the portfolio (usually eight)

**Note: Monthly portfolio returns were subject to different constraints as displayed under section 5.3*

The mean portfolio return over the investment period was calculated by taking the average of all the monthly portfolio returns, subject to a select weighting, over the investment period.

This is set out in the Equation 12 below:

$$\mu_p = \left(\frac{1}{n}\right) \sum_{m=1}^n R_{pm} \quad (12)$$

where

μ_p is the mean portfolio return over the investment period,

R_{pm} is the m^{th} monthly portfolio return,

n is the number of months.

Step 3

The portfolio mean semivariance was calculated through the implementation of equation 8 stated on page 34, but also presented on page 37 below. This equation first determines the mean semivariance through the portfolio returns deviation from its benchmark return. The mean semideviation was then calculated by square rooting the semivariance.

The benchmark return is normally implemented as either zero or the mean return of the portfolio under analysis. However, for this study, the STeFI+3% was chosen as the benchmark for the portfolio. This index is an industry benchmark for many actively managed funds, and provides a real-world input to the calculation of semivariance.

Only returns below the benchmark, in other words, negative deviations, are included in the calculation of semivariance and, in turn, semi-deviation (hence the minimum, or Min, function below).

$$\psi_{pB}^2 = \left(\frac{1}{n}\right) \sum_{m=1}^n [\text{Min}(R_{pm} - B, 0)]^2 \quad (8)$$

where

ψ_{pB}^2 is the mean portfolio semivariance over the full investment period,

R_{pm} is the monthly portfolio return for m^{th} month,

B is the benchmark return (in this study the STeFI+3%),

n is the number of negative deviations $[0 > (R_{pm} - B)]$ subject to the constraint:

$$-1 < R_{pm} - B < 0$$

Benchmark calculation

STeFI+3% was calculated by taking the average of the monthly STeFI index returns over the investment period and adding the effective monthly rate of 3% a year to it.

Step 4

It is then possible to optimise this portfolio through the optimisation of its risk-return efficiency. The Sortino ratio considers the downside risk adjusted return with regards to the standard deviation of negative asset returns. This Sortino ratio applies the semideviation of negative deviations from its benchmark (still a downside risk). This is the risk-return optimisation we will be looking at. From Equations 8 and 12 a risk-return measure can be set up as shown below (Equation 13):

$$Eff = \frac{\mu_p - r_f}{\sqrt{\psi_{pB}^2}} \quad (13)$$

Where

Eff is the portfolio's risk-return efficiency (Sortino ratio)

μ_p is the mean portfolio return over the investment period,

r_f is the risk-free rate,

$\sqrt{\psi_{pB}^2}$ is the mean semideviation of the portfolio over the investment period.

The proxy used for the risk-free rate was the South African government bond, the R186. The risk-free rate was calculated by first taking the average yield per annum of the R186 over the investment period and then calculating the effective monthly rate of the annual yield.

Step 5

Using Equations 8, 12 and 13, the portfolio mean monthly semivariances, semideviations, returns and risk-return efficiencies were calculated through optimisation. In total, 20 000 random asset weighted scenarios were generated, forming 20 000 random portfolio weight combinations. The result was 20 000 portfolio's mean monthly semivariances, semideviations and returns, which were used to solve for the highest efficiency portfolio. Once the highest efficiency was found, these weightings were used as the base weightings to run the solver function for maximising Equation 13 (the efficient ratio), subject to specific weighting constraints and the portfolio framework described in Section 5.3 on page 39. This process was repeated five times to make sure that the correct optimal portfolio was found.

Firstly, this step was done for a well-diversified South African portfolio consisting of domestic equity, domestic bonds, domestic real estate, domestic cash, foreign bonds and foreign equity (*i.e.*, six asset classes), but with no holdings in Bitcoin or gold bullion. This is considered to be the base portfolio. Next the process was repeated with the inclusion of gold bullion as the seventh asset classes and, thirdly, the process was run again with the inclusion of Bitcoin over and above the gold bullion inclusion (taking the portfolio up to eight asset classes). This process was repeated for all portfolio frameworks. Finally, Bitcoin and gold bullion were added separately to the base portfolio in order to compare their diversification and asset allocation effects on the portfolio. The practical long-only portfolio as constrained under Regulation 28 of the South African Pension Fund Act was the portfolio framework used.

Step 6

The efficient frontier was constructed by solving for the following portfolios;

1. The minimum mean monthly semideviation portfolio
2. The most optimal risk-return efficient portfolio (*i.e.*, highest Sortino ratio)
3. The maximum mean monthly return portfolio

These portfolios were calculated using the same process as above. The optimal risk-return efficiency portfolio was explained above. For Portfolios 1 and 3 above, 20 000 random portfolio weights were generated, for which 20 000 portfolios mean monthly semivariances, semideviations and returns were calculated. Once calculated, the weightings of the relevant portfolio with the lowest mean monthly semideviation and the highest mean monthly return were used as base weights. These base weights were used in the solver function to minimise Equation 8 and maximise Equation 12, subject to the weighting and portfolio framework constraints explained in Section 5.3. This process was also repeated five times to make sure the most optimal portfolios were found.

5.3 Portfolio Strategy

In order to evaluate the diversification benefits of Bitcoin under the mean-semivariance approach, the perspective of a South African investor was adopted to construct well-diversified portfolios. The proxies for domestic equity, domestic bonds, domestic property, domestic money market, foreign bonds and foreign equity, as explained in Chapter 4, were used to optimise the portfolio's risk-return efficiency. The study started with the base portfolio as stated above, to which was then added gold bullion and Bitcoin assets one at a time under different portfolio frameworks. For a direct comparison of Bitcoin and gold bullion as diversification assets, Bitcoin and gold bullion were added separately to the well-diversified base South African portfolio with no overlapping additions.

In total, four portfolio frameworks were tested - three theoretical and one practical framework. The practical framework is set under a constrained portfolio, where the constraints are those that a South African pension fund manager would face as required by Regulation 28.

5.3.1. Regulation 28

Regulation 28 of South Africa's Pension Fund Act was used as the constraint imposed on the long only constrained portfolio on this study. The revised Regulation 28 of the Pension Fund Act came into existence on the 1st of July 2011, and imposes restrictions on the maximum percentage of different asset classes that pension funds may hold. The objective of Regulation 28 is to ensure that pension funds act in line with their fiduciary duty in the best interest of their members, and specifically to ensure prudent investment decisions in line with risk, return and diversification needs. The constraints are enacted on all varieties of asset classes, namely equity

(domestic and foreign), property (domestic and foreign), foreign assets, commodities and alternative assets. Constraints on the fund are based on the aggregated fair value of the assets a pension fund holds.

Regulation 28 asset class constraints are as follows:

- A maximum of 100% in cash. However, only 25% of the cash holding may be on deposit with an individual bank, and only 5% with a foreign bank.
- A maximum of 100% in debt instruments issued by or guaranteed by the Republic of South Africa, otherwise 10% in debt instruments issued by or guaranteed by the government of a foreign country.
- A maximum of 75% in equities (domestic and foreign)
- A maximum of 25% in property (domestic and foreign)
- A maximum of 25% in foreign assets (in this case equities and bonds)
- A maximum 10% in the Gold commodities, otherwise 5% in other commodities
- A maximum of 15% in alternative assets. Hedge funds and private equity are limited to 10% each, while other assets not referred to are limited to 2.5%.

Considering that cryptocurrencies are not specifically mentioned in Regulation 28, Bitcoin would be considered an “asset not referred to”, which implies a 2.5% portfolio weight constraint for it.

5.3.2. Portfolio framework

The risk-return efficiency of the optimal portfolio is subject to four different optimisation frameworks namely; equally-weighted, long-only unconstrained, long-only constrained and +100%/-100% unconstrained.

5.3.2.1 Theoretical Frameworks

Framework 1: Equally-Weighted ($w_i = 1/N$)

The first framework, unlike the following three, does not rely on the optimisation of the risk-return/efficiency ratio to find the optimal portfolio asset weights (w_i). In this framework, the portfolio consists of equally-weighted assets where each asset's weight in the portfolio is $1/N$ (N is the number of assets that are available to make up the optimal portfolio), and thus no

optimisation process is needed. As stated by deMiguel et al. (2007), the equally-weighted framework can lead to comparable or higher Sharpe ratios when compared against different portfolio optimisation frameworks. This is due to the weak predictive power of many standard risk and return measures. For this reason, the equally-weighted framework was included in this study to see if this framework can beat the predictive power of the chosen risk-return measure (mean-semivariance) under various optimisation frameworks. This framework provides an unbiased view of the raw effect of Bitcoin's diversification power.

Framework 2: Long-Only Unconstrained ($w_i \in \mathbb{R}: 0 \leq w_i \leq 1$)

In this framework, a short selling constraint is imposed on the assets, but without a weight allocation constraint. The short selling constraint is imposed to better reflect the effect Bitcoin will have for a long only portfolio manager with minimum weighting mandates. This allows a single asset (Bitcoin) to go over the standard 10% maximum weighting limit in one so called 'stock', a 5% weight in a commodity (cryptocurrency) and a 10% weight in the gold commodity (Bitcoins comparison). This framework limits the sum of all asset class' weights to 100%.

Framework 3: +100%/-100% Unconstrained ($w_i \in \mathbb{R}: -1 \leq w_i \leq 1$)

In this framework, the short selling constraint was relaxed and the asset weights were allowed to move between +100% and -100%. This framework was intended to give an insight into what Bitcoin's effect may be under a hedge fund-like framework, given that hedge funds in South Africa are not subject to Regulation 28, or short-selling constraints. Again, this framework does not constrain any asset weightings, but does limit the sum of all asset class' weights to 100%.

5.3.2.2 Practical Framework

Framework 4: Long-Only Constrained ($w_i \in \mathbb{R}: 0 \leq w_i \leq 1$) (Subject to Regulation 28)

In this framework, the Regulation 28 constraints are imposed on the portfolio described above. Bitcoin is seen as its own asset class, and subjected to Regulation 28's 5% (other commodity) and 10% (gold commodity) weighting constraint. The reason for the latter is that under a 10% constraint Bitcoin can be directly compared to gold in terms of its diversification abilities. It was thought appropriate to compare the diversification properties of these two assets under the

same constraints (in this case the Regulation 28 constraint on gold). The implicit assumption is that if Bitcoin was to be included in Regulation 28, it would be under an alternative asset (15% limit in Regulation 28), but due to its excessive risk, it would likely be reduced to a 10% holding limit or a commodity constraint such as gold. Alternatively, Bitcoin could be compared to an “other commodity”, in which case it could be limited to a 5% level. Regulation 28 also refers to “other assets not referred to in this schedule” at a 2.5% weighting constraint. In reality, given Bitcoin’s currently undefined status in terms of Regulation 28, the 2.5% constraint is probably the most accurate view of Bitcoin’s current definition under Regulation 28. The 2.5% constraint was analysed when Bitcoin and gold were directly compared. This framework gives the most accurate and real world description of Bitcoin’s diversification abilities and enhancement characteristics. This framework limits the sum of all asset classes’ weights to 100%.

The next chapter will discuss the findings of this study.

Chapter 6: Results and analysis

This chapter presents the results and analysis of this study, based on the tests performed as described in Chapter 5. These tests seek to help in the understanding of Bitcoin's attributes as a diversifier to a South African investor. Section 6.1 shows the results of a preliminary analysis of the data obtained. This is followed by Section 6.2, which presents a detailed discussion on the full results obtained.

6.1 Explanatory and descriptive data analysis

Under this section an analysis is conducted to assess the distribution of the data being used in the relevant tests. It also shows the relevant graphing of the data for further insight into the return characteristics and trends.

Key statistics

Table 6.1: Descriptive statistics

	Bitcoin (Rands)	SA Bonds (ALBITR)	SA All share (JALSHTR)	SA Property (JSAPY)	STeFI (Cash)	Global developed bonds (Rands)	Gold Bullion (Rands)	MSCI World (Rands)
Mean	0,2477	0,0074	0,0110	0,0111	0,0051	0,0074	0,0078	0,0130
Median	0,1052	0,0076	0,0101	0,0160	0,0049	0,0026	0,0120	0,0086
Standard deviation	0,6837	0,0215	0,0322	0,0387	0,0007	0,0387	0,0519	0,0390
Semideviation	0,1853	0,0219	0,0288	0,0462	0,0026	0,0351	0,0538	0,0328
Skewness	3,9070	-0,2012	0,3624	-0,7469	0,4006	0,4298	0,0459	0,3491
Excess Kurtosis	19,5156	1,2453	-0,2198	1,1071	-0,8377	0,8413	0,4725	0,6654
Range	4,9586	0,1316	0,1505	0,2057	0,0028	0,2291	0,2067	0,3014
Maximum	4,5803	0,0649	0,0935	0,0948	0,0067	0,1329	0,1724	0,1220
Minimum	-0,3784	-0,0667	-0,0570	-0,1109	0,0039	-0,0962	-0,1290	-0,0847
# Observations	91	91	91	91	91	91	91	91

This table displays the key statistics for monthly returns of all relevant asset classes used in the optimisation framework, 2010-2018

From Table 6.1 it can be seen that Bitcoin exhibits the most extreme values and descriptive statistics of all the included assets over the period August 2010 to February 2018. For example, Bitcoin has the largest mean monthly return (24.77%), with the local property and All Share

indices coming a distant second and third with mean monthly returns of 1.11% and 1.10%, respectively.

Bitcoin's monthly returns also have a significantly larger standard deviation (68.37%) and range. This, together with a larger maximum value observed (458.03%) than all other asset classes, suggest that Bitcoin was not only a much riskier asset over the sample period in general, but especially so when upside swings are considered. For this reason, the mean-semivariance approach, which captures the semivariance of an asset class and calculates the semideviation (downside risk) of that asset, was used in this study. Although Bitcoin's semideviation is still high (18.53%), compared to the other asset classes, it exhibits a significant reduction between its standard deviation to its semideviation (from 68.37% to 18.53%). Thus, Bitcoin has less of a deviation risk under the semideviation framework than the standard deviation framework. However, gold bullion had the second highest semideviation (5.38%) and the fifth highest mean return (0.78%). This suggests that its effect on enhancing the optimal portfolio's efficiency (in the form of the Sortino ratio) would be low in comparison.

Table 6.1 also displays all the asset classes' skewness and excess kurtosis. As expected and stated in the data section (Chapter 4) Bitcoin exhibits high excess kurtosis and skew. The excess kurtosis is measured relative to that of the normal distribution, which has an excess kurtosis of zero, but allows for a range of -0.5 to 0.5. Bitcoin's return distribution appears to be excessively positively skewed (3.907). Positive skewness like this is desirable to investors, as it shows that large movements in the asset price is likely to result in a gain rather than a loss, and that the asset has the tendency to have returns above the mean return. Bitcoin's return distribution also displays a large excess kurtosis (19.52). This means that Bitcoin's return distribution has a much steeper peak and fatter tails than a normal distribution. These fatter tails show that majority of returns lie further away from the mean-return. These observations are in line with previous academic literature¹². It is important to note, though, that Bitcoin is a very recent asset class, and that much of its history to date is indicative of a speculative bull phase, which may not be representative of its future path.

¹² See Baur, Lee and Hong (2015); Yermack (2013); Burniske and White (2016); Wu and Pandey (2014); Brière, Oosterlinck and Szafarz (2015); and Eisl, Gasser and Weinmayer (2015)

Table 6.2: Correlation Matrix

	Bitcoin (Rands)	SA Bonds (ALBITR)	SA All share (JALSHTR)	SA Property (JSAPY)	STeFI (Cash)	Global developed bonds (Rands)	MSCI World (Rands)	Gold Bullion (Rands)	USDZAR
Bitcoin (Rands)	1,0000								
SA Bonds (ALBITR)	-0,0952	1,0000							
SA All share (JALSHTR)	-0,0335	0,0952	1,0000						
SA Property (JSAPY)	-0,0464	0,5026	0,1884	1,0000					
STeFI (Cash)	-0,1757	-0,0248	-0,0407	-0,0692	1,0000				
Global developed bonds (Rands)	0,0411	-0,5010	-0,1476	-0,3673	-0,0775	1,0000			
MSCI World (Rands)	0,1264	-0,5149	0,3798	-0,2368	-0,0910	0,5762	1,0000		
Gold Bullion (Rands)	-0,0300	-0,1429	0,1129	-0,2372	0,0197	0,5713	0,2339	1,0000	
USDZAR	0,0077	-0,5905	-0,2063	-0,4264	-0,0821	0,9403	0,6189	0,4472	1,0000

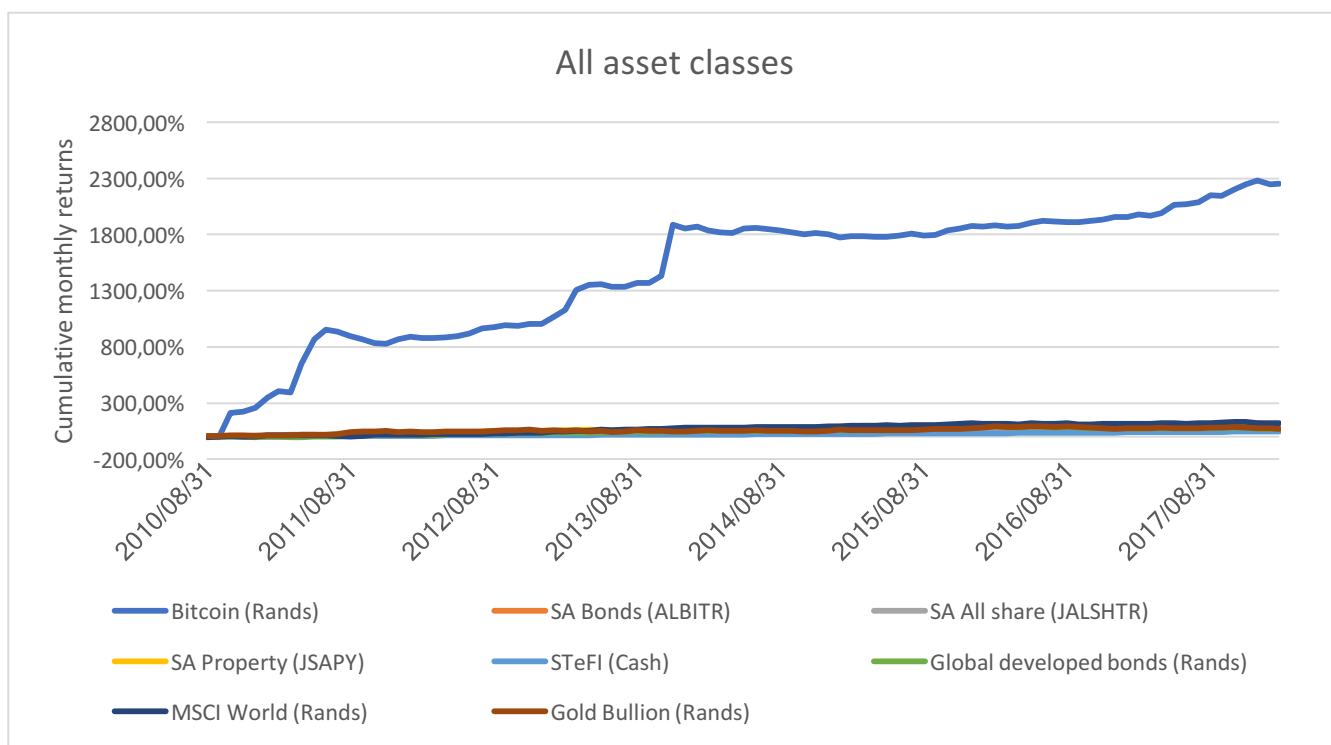
This table displays the correlation coefficients of the assets displayed in our portfolio, along with Bitcoin's Rand effect

Table 6.2 shows that Bitcoin has either a close to zero or a negative correlation with most asset classes available to a South African investor. The negative correlations between Bitcoin and all the direct South African asset classes show empirical evidence for its diversifying abilities, with the largest negative correlation being with cash (-0.18). The two largest positive correlations and therefore lower evidence for Bitcoin's diversification ability with them, comes from foreign bonds and equity. Bitcoin also shows a negative correlation with gold (-0.03), implying that both asset classes could possibly be used together to further diversify a portfolio. Bitcoin seems to be positively correlated to foreign assets, which is in line with prior studies¹³. The USDZAR currency cross shows that Bitcoin has close to zero correlation with South African currency returns. This is in line with the study conducted by Burniske and White (2016), who found that emerging market currencies had the lowest correlations with Bitcoin. The low currency correlation may cause the foreign assets' rand returns to be less correlated than its US dollar counterparts, thus supplying a further reason to test the enhancing portfolio characteristics of Bitcoin.

As stated by Brière et al. (2015), correlations can change vastly under stress, and only capturing the correlations in a bull market could skew the results of such correlations. This is a potential limitation of this study, which does, however, include the bear trend that Bitcoin has been in since December 2017 (Appendix A1.4).

¹³ See Yermack (2013), Brière et al. (2015) and Eisl et al. (2015)

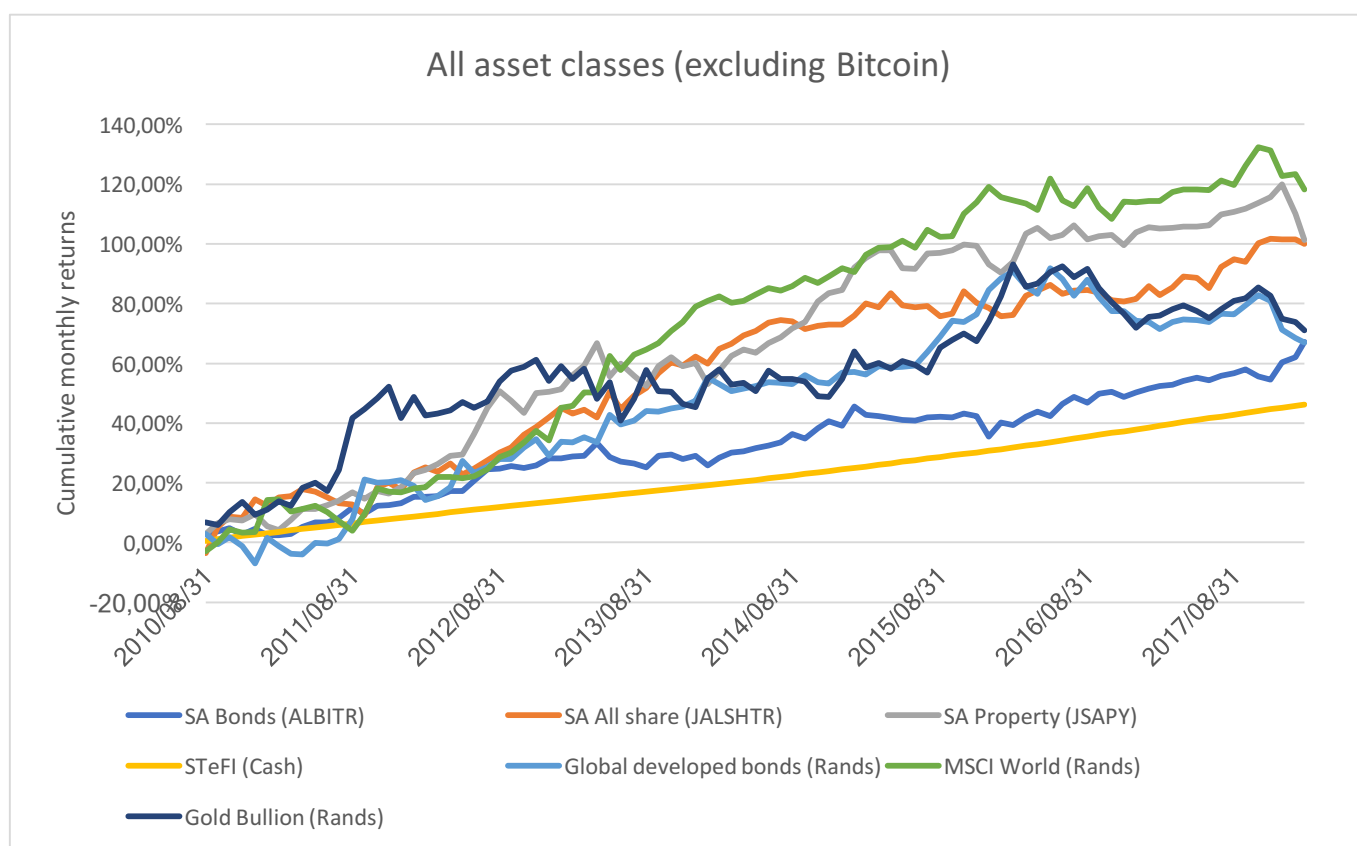
Figure 6.1: Asset classes' cumulative monthly returns



This figure displays the cumulative returns of all the asset classes, over the investment period.

Figure 6.1 shows the cumulative returns of all asset classes over the investment period. It is clear to see that the cumulated return in Bitcoin far surpasses that of the traditional asset classes over the period in question. This return was driven by a mass price rally over the period (see Appendix A1.4), resulting in cumulated returns over the period of 2253.92%. This large cumulated return comes with very large risk (measured as semideviation), as can be seen in Table 6.1 above.

Figure 6.2: Asset classes' cumulative monthly returns



This figure displays the cumulated returns of all the asset classes, excluding Bitcoin, over the investment period.

The figure above shows the cumulative performance of all the asset classes under study, except for Bitcoin. Over the period under investigation, the MSCI World Index, SA Property and SA All Share Index are the top three returning assets, respectively. Gold's returns declined significantly from mid- to late 2016 through to the end of the sample period (February 2018) (see Appendix A1.7). This was driven by both the gold price declining, and rand depreciation, causing the ZAR gold price to drop. From this we can see that the MSCI World Index (118.20%), SA listed property (101.27%) and SA equities (100.00%) would have likely been the main drivers of portfolio returns, over the investment period, when Bitcoin was not included. The cumulative returns graph above shows the highest returning asset classes, but not necessarily the highest weighted asset classes within the optimal portfolios not containing Bitcoin.

It was found that global equity, South African bonds and South African listed property were the three main holdings in the portfolio in order of weighting when Bitcoin was not included.

Thus, the cumulative returns graph above shows the highest returning asset classes, but not necessarily the highest weighted asset classes, in the optimal portfolios not containing Bitcoin. Global equity was the highest weighted asset in all the portfolios that were unconstrained, and was capped at the full 25% foreign exposure allowed by Regulation 28. South African equity was present at a higher weighting in the constrained portfolios, mainly due to global equity being capped.

6.2 Portfolio results and analysis

This section will take an in-depth analysis of the results from this study. The section is laid out as follows: Analysis into portfolio optimization under different frameworks (6.2.1), Analysis of the efficient frontiers under different frameworks (6.2.2) and the direct comparison of Bitcoin and gold bullion under different frameworks (6.2.3)

6.2.1 Analysis into portfolio optimisation under different frameworks

This section discusses the empirical results of Bitcoin's addition into a portfolio under the different portfolio frameworks, as mentioned in the methodology chapter (Chapter 5). The portfolio optimisation started out with the base portfolio (the six traditional asset classes, with no allocation to either Bitcoin or gold), then in the next iteration gold was added to the portfolio (still no Bitcoin), followed by the addition of Bitcoin on top of the inclusion of gold, and finally, Bitcoin was constrained to (a) a 5% and (b) a 10% weighting within the portfolio, with gold still being included.

Table 6.3: Portfolio optimisation results

Portfolio Optimization Framework	Mean Monthly Optimal Bitcoin Weight	Mean Monthly Semideviation	Mean Monthly Return	Mean Monthly Efficiency (Sortino ration)
Equally-Weighted (Bitcoin)	12,50%	2,44%	3,88%	1,317
Equally-Weighted (No Bitcoin)	0,00%	1,60%	0,90%	0,147
Equally-Weighted (No Bitcoin or Gold)	0,00%	1,26%	0,92%	0,197
Long-Only Unconstrained (Bitcoin)	12,74%	2,07%	3,97%	1,597
Long-Only Unconstrained (No Bitcoin)	0,00%	1,31%	1,05%	0,287
Long-Only Unconstrained (No Bitcoin or Gold)	0,00%	1,30%	1,04%	0,287
Long-Only Constrained (Bitcoin 5%)	5,00%	0,92%	1,94%	1,367
Long-Only Constrained (Bitcoin 10%)	10,00%	1,67%	3,29%	1,577
Long-Only Constrained (No Bitcoin)	0,00%	1,31%	1,02%	0,267
Long-Only Constrained (No Bitcoin or Gold)	0,00%	1,35%	1,03%	0,267
*+100%/-100% Unconstrained (Bitcoin)	13,95%	2,23%	4,35%	1,657
*+100%/-100% Unconstrained (No Bitcoin)	0,00%	2,47%	1,60%	0,377
*+100%/-100% Unconstrained (No Bitcoin or Gold)	0,00%	2,51%	1,60%	0,377

This table displays the results of each optimising portfolio under different portfolio frameworks. All data is presented as monthly means over the investment period (July 2010 to February 2018).

Key to Table 6.3

- *No Bitcoin or gold-* This is the base portfolio, with no Bitcoin or gold included in the optimisation.
- *No Bitcoin-* Gold is added to the optimisation, but not Bitcoin
- *Bitcoin* – Bitcoin is added to the optimisation which already includes gold
- *Bitcoin 5%* - All assets are constrained to their Regulation 28 limits, with Bitcoin constrained to a 5% weighting, and gold constrained to a 10%
- *Bitcoin 10%*- All assets are subject to Regulation 28, with Bitcoin constrained to a 5% weighting, and gold remains constrained to 10% as above.

Table 6.3 shows, for all four portfolio frameworks over the investment period of 91 months, Bitcoin's mean monthly portfolio weightings, mean monthly semideviations, mean monthly returns, and mean monthly risk-return efficiencies, as measured by Sortino ratios.

From the above, we see that all portfolio frameworks' reactions are similar, in that upon the inclusion of Bitcoin into the portfolio, the risk-return efficiency ratio is enhanced. Bitcoin's weight remained relatively high within the limits imposed by the relevant constraint, in line

with the findings of Carpenter (2016). The largest inclusion of Bitcoin (13.95%), as expected, comes under the +100%/-100% unconstrained portfolio. This is understandable due to the absence of constraints on its weighting and direction of weighting. This is in contrast to the findings of Eisl et al. (2015) of the lowest Bitcoin weighting under the +100%/-100% portfolio constraint. However, Eisl et al's (2015) methodology involved the constraining of the asset class weightings for the +100%/-100% portfolio, unlike the unconstrained portfolio weighting framework used in the present study. A further possible reason for this difference is that Bitcoin's risk can possibly be better diversified away in the South African context due to its negative correlations with most South African asset classes (see Table 6.2).

On average over all the portfolio frameworks, the inclusion of Bitcoin increased mean semideviation by only 0.27%. The biggest increase was found in the equally-weighted portfolio, with an increase of 0.84% (from 1.60% to 2.44%). This is understandable due to the equally-weighted framework's lack of risk-return optimisation. These findings of increased risk upon addition of Bitcoin are consistent with the findings of Eisl et al. (2015), Brière et al. (2015) and Carpenter (2016). Bitcoin's inclusion under the constrained framework at the 5% level reduced the semideviation to 0.92%. This is due to Bitcoin's massive returns being offset by high holdings in cash. Cash is the lowest semideviation asset (0.3%) with the most negative correlation to Bitcoin (-0.18) (see Tables 6.1 and 6.2, respectively). Bitcoin's inclusion under all frameworks also dramatically increased mean monthly returns by 2.37% on average. This return enhancement was again most noticeable in the equally-weighted portfolio, with a 2.98% increase, from 0.9% to 3.88%. This increase in return was due to this portfolio increasing its risk (semideviation) the most, as stated above. However, increases in returns from the long-only unconstrained and the +100%/-100% unconstrained portfolios were not far behind, with return increases of 2.92% and 2.76%, respectively.

From Table 6.3 on page 49, it can be seen that, under all portfolio optimisation frameworks, the risk-return/efficiency ratio (the Sortino ratio) is increased dramatically with the inclusion of Bitcoin. The most efficient portfolio is the +100%/-100% unconstrained portfolio, with an efficiency (Sortino ratio) of 1.65, whereas the long-only unconstrained and long-only constrained to 10% Bitcoin had Sortino ratios of 1.59 and 1.57, respectively. Even the relatively simple and naïve equally-weighted framework had an efficiency ratio of 1.32, which understandably was the smallest efficiency due this portfolio not being optimised. As stated under the portfolio framework description, the equally-weighted framework provided an

unbiased comparable view. The equally-weighted efficiency is the lowest of all frameworks and shows that, although comparable, the mean-variance risk-return measures show higher predictive power under optimisation techniques. The large increases in efficiency indicate Bitcoin's great ability to enhance a South African portfolio. These findings are consistent with the findings of Eisl et al. (2015), Brière et al. (2015) and Carpenter (2016).

In terms of asset allocation, South African bonds had the second highest weighing in portfolios on average when Bitcoin was excluded, and the highest weighting on average when Bitcoin was included. This was because South African bonds had the strongest negative correlation to the MSCI World Index (-0.51) and the largest negative correlation to Bitcoin of -0.095 (see Table 6.2), as well as the second lowest mean monthly semideviation of 2.19%. These characteristics help alleviate the risk attached to the large weightings of the MSCI World Index and Bitcoin. Thus, South African bonds function to alleviate the risk of the high return asset classes to enhance the efficiency of the portfolio, more than to add outright return enhancements. Similarly, South African cash also alleviates the portfolio risk introduced by Bitcoin, due to it being the lowest risk asset (semideviation of 0.26%), and having the strongest negative correlation with Bitcoin (-0.1757).

The risk and return enhancements discussed are similar to the findings of the studies discussed above, but seems to show a lowering of risk (semideviation) under some portfolio frameworks. For example, under the long-only constrained (to 5% Bitcoin) portfolio framework Bitcoin's inclusion reduces risk (semideviation) from 1.31% to 0.92%. This was due to an extensive weighting in the SA cash asset class, which offsets Bitcoin's high returns. As shown in Table 6.1 and Table 6.2, the cash asset class had the lowest semideviation (0.26%), but the strongest negative correlation with Bitcoin (-0.18). The lowest Bitcoin weighing (5%) allows for Bitcoin to reduce its semideviation impact on the portfolio by diversifying out that risk against the cash asset class. Risk was also diversified out with a high weighting in the SA bonds asset class, which has the second strongest negative correlation to Bitcoin (-0.095) and second lowest semideviation (2.19%) (see Table 6.1). However, this option is not as efficient when higher weightings of Bitcoin are used (the long-only constrained to 10%) in the portfolio. Under the +100%/-100% unconstrained portfolio there is a reduction in semideviation due a 100% short position in cash not being taken to fund return enhancements elsewhere, but rather the addition of Bitcoin to fund those returns with higher weightings in the stronger negatively correlated returns such as the ALBI and a smaller short position in cash.

This finding shows that Bitcoin, when used in a South African investment context, can in fact not only increase all risk-return/efficiency ratios (Sortino ratios), but can reduce risk under some portfolio frameworks especially at lower levels of Bitcoin. Importantly, Bitcoin reduces risk under the Regulation 28 portfolio when weighted at 5%, whilst enhancing the efficiency.

The difference from the findings of past studies could be due to past studies all taking a developed market (mainly US) view. In the developed market view, while correlations were low, as shown by Eisl et al. (2015) and Brière et al. (2015), very few, if any, asset returns were negatively correlated with Bitcoin returns. As shown in the correlation matrix in Table 6.2, South African assets are all to some extent negatively correlated with Bitcoin. This causes Bitcoin to reduce the semideviation when added to a South African portfolio, despite its extremely large semideviation as shown in Table 6.1.

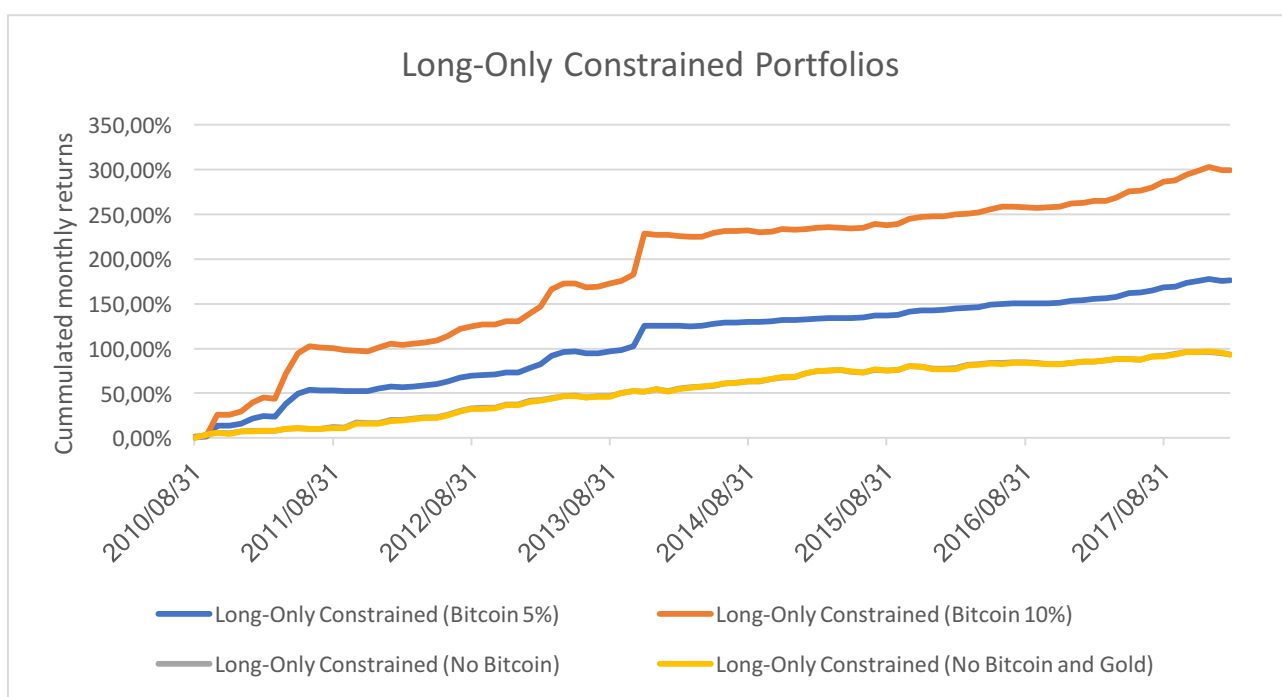
In this study the inclusion of Bitcoin in the fully diversified portfolio (including gold bullion), under all frameworks, resulted in the asset allocation being reduced in local equity, local property and foreign equity. Thus, the All Share Index, the South African Property Index and the MSCI World Index, were the most reduced asset weightings, in that order. This is due to Bitcoin's returns being associated with even larger risk. Thus, the higher risk asset classes were reduced to allow for lower risk and higher negatively correlated assets to increase their weighting in the portfolio.

Table 6.3 gives an insight into the impact of gold on the same portfolio. In none of the optimised portfolios does gold reach its Regulation 28 limit of 10%. Thus, gold's weighting, for portfolios where Bitcoin was not included was, on average, 2.86%, which further reduced to 1.60% when Bitcoin was included (excluding of course the equally-weighted portfolios). Therefore, in all cases, except in the equally-weighted portfolio, gold increased the efficiency (Sortino ratio) of the portfolios only slightly. These results are in line with those of Baur and McDermott (2010), who showed that gold is a weak hedge in emerging markets, and Chua et al. (1990), who found that gold bullion increased the risk/return ratio of a portfolio. Chua et al. further showed that gold has better diversification abilities over short-term time horizons. From Table 6.3 it is clear that, although gold increases the efficiency (Sortino ratio) of the South African portfolio, it does not compare to the increase in efficiency when Bitcoin is added over and above gold. This is due to the risk attached to gold being too high for the return over the period. Table 6.1 shows that gold had the second highest semideviation (0.054), after that of

Bitcoin (0.185), but only the 5th highest mean return of 0.008 (0.8%). Gold's diversification benefit is due to its relatively low correlations with most of the asset classes in question.

The cumulated monthly returns of the optimal portfolios are graphed under the equally-weighted, long-only unconstrained and +100%/-100% figures in Appendices 1.15 to 1.17. These figures show that the addition of Bitcoin to the portfolios increase their returns dramatically. Under the equally-weighted, long-only unconstrained, and +100%/-100% portfolios, Bitcoin increases the cumulative portfolio return to 353.09%, 361.43% and 396.2%, respectively, over the investment period. These results are in line with the findings of Eisl et al. (2015) and Carpenter (2016), who both found that Bitcoin's inclusion vastly increases cumulative portfolio returns, but that the inclusion of gold in these portfolios makes a minimal difference to the cumulated returns over the investment period.

Figure 6.3: Long-only constrained portfolios cumulated returns

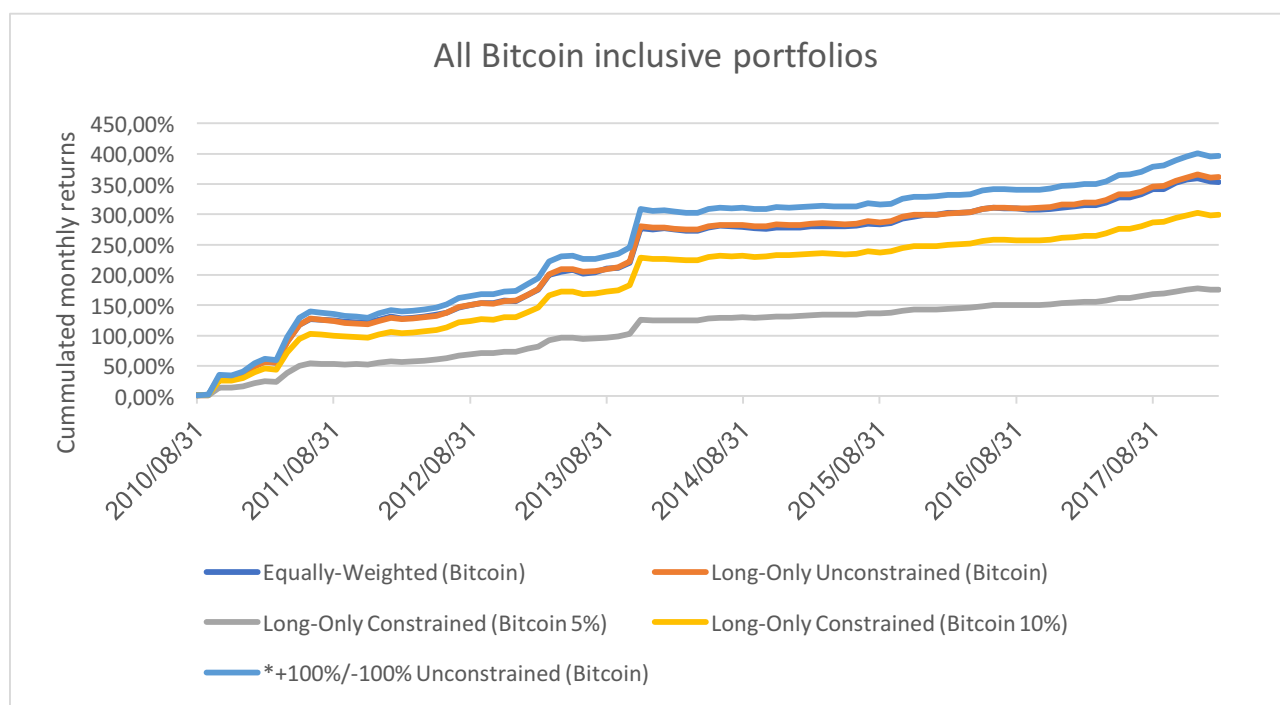


This figure displays the cumulated returns of the four variations of long-only constrained optimal portfolios over the investment period. Returns are kept at constant optimal weighting over the investment period, and re-weighted every month.

Figure 6.3 above shows the effect of constraining the addition of Bitcoin to the portfolio. The highest return relates to the 10% constrained portfolio (299.40%), and the second highest to the 5% constrained portfolio (176.15%). This is to be expected due to Bitcoin's large price

increase over the investment period (see Appendix A1.4). Figure 6.3 illustrates the almost negligible effect of gold's addition to the optimally constrained South African portfolio. The Long-only constrained (No Bitcoin) line can barely be seen in figure 6.3 as it sits behind and closely tracks the Long-only constrained (No Bitcoin and gold) line. Given the fact that Golds has the second highest risk (semideviation) attached to it, and its correlation to the main driver, the MSCI World Index, was 0.234 (see Table 6.2), the portfolio contained a small weighting (3.5%) in gold, as better return enhancers could not be found with lower correlations and less risk.

Figure 6.4: All Bitcoin inclusive portfolios' cumulated returns



This figure displays the cumulated returns of the five variations of optimal portfolios that included Bitcoin over the investment period. Returns are kept at constant optimal weighting over the investment period, and re-weighted every month

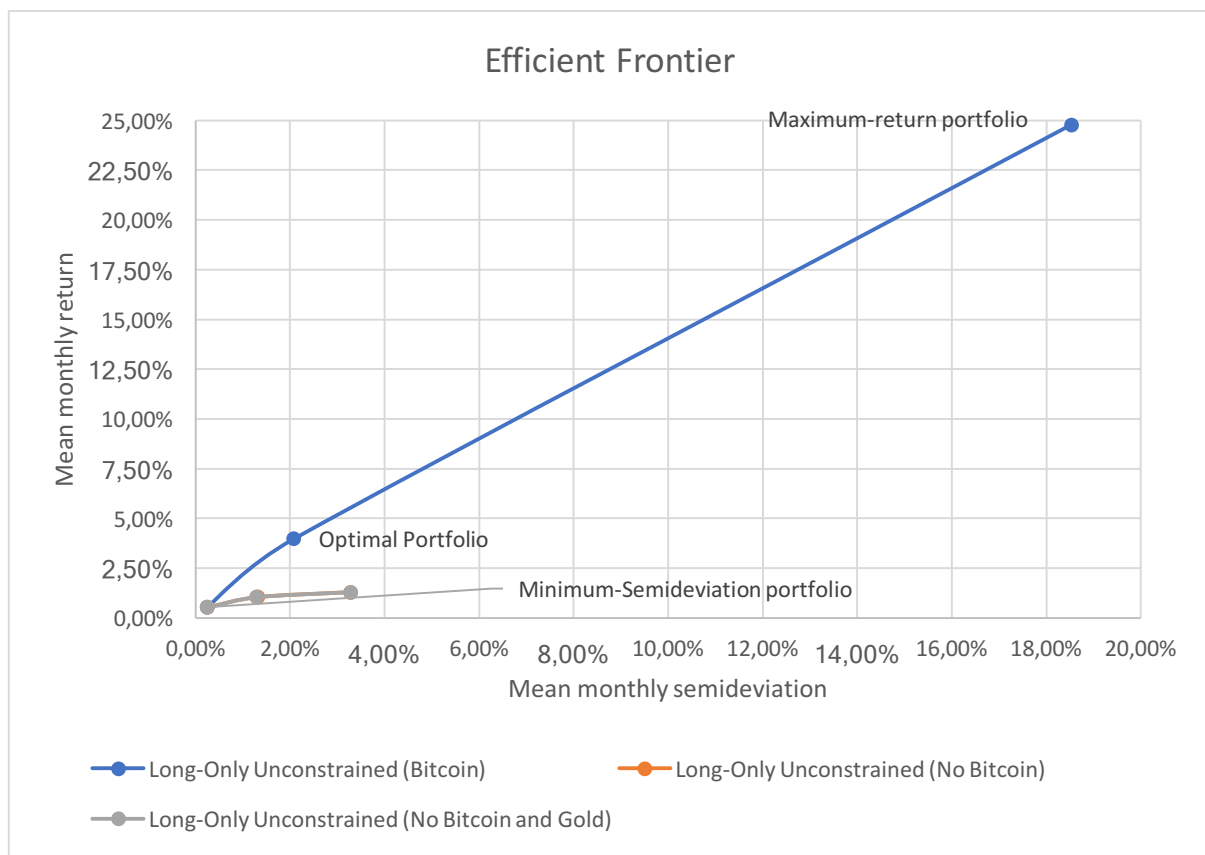
Figure 6.4 shows all optimal portfolios that include Bitcoin. As expected, the +100%/-100% portfolio returned the highest cumulative return over the investment period (396.2%). The ability to go short and have no constraint on weighting reflects the ability of Bitcoin to diversify a portfolio for ultimate efficiency. Also, as expected, constraining Bitcoin to a 5% weighting (the lowest weighting) resulted in the lowest cumulated return over the investment period (175.15%). Furthermore, the lower the weighting in Bitcoin was, the smoother the returns were over the period. The long-only constrained portfolio with Bitcoin constrained to 5% has a

much smoother return than all its counterparts, and it has the lowest semideviation (see Table 6.3).

6.2.2 Analysis of the efficient frontiers under different frameworks

This section considers the effect that Bitcoin has on the efficient frontiers of various optimal portfolios under different portfolio frameworks. To be able to compare the effects, long-only unconstrained and long-only constrained portfolios are used in this analysis.

Figure 6.5: The efficient frontier of unconstrained long-only portfolios



This figure above displays the efficient frontier shift that takes place upon the inclusion of Bitcoin and gold in optimal portfolios.

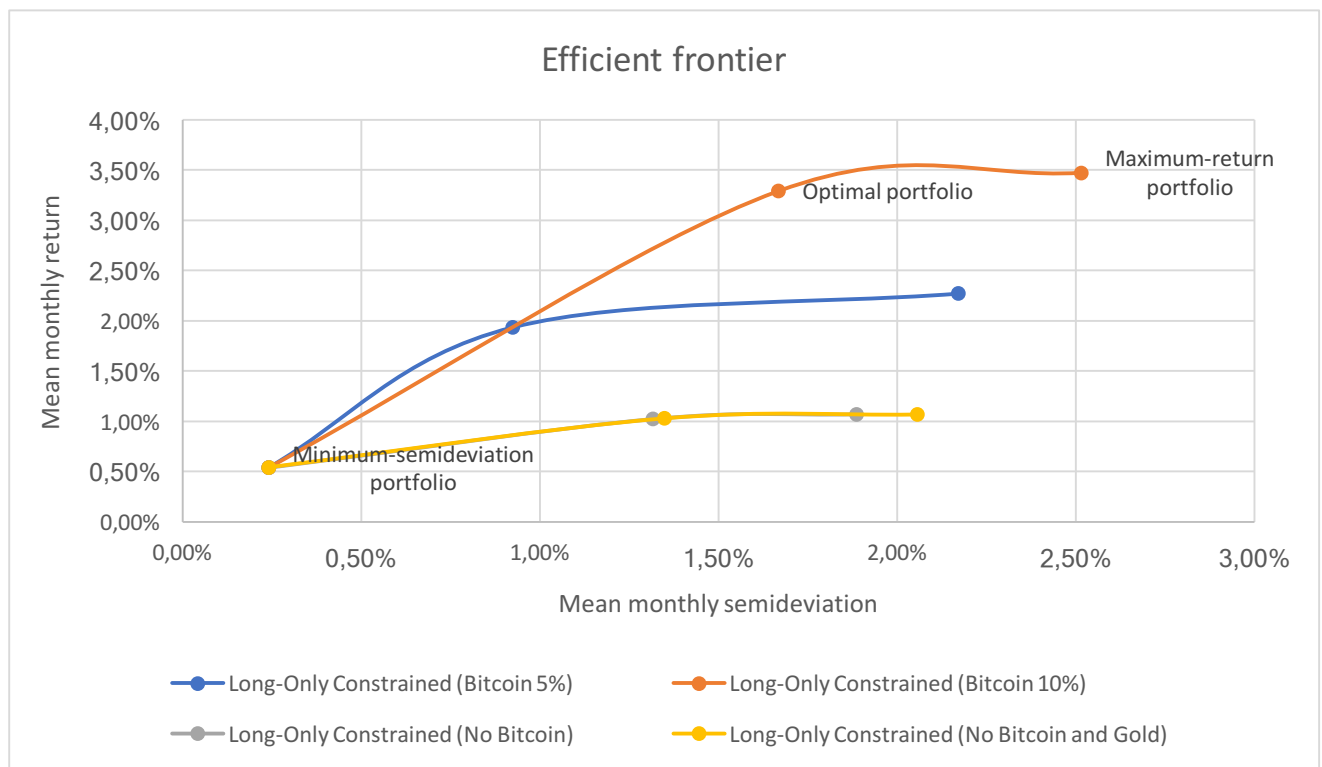
Figure 6.5 shows the long-only unconstrained efficient frontier shift when Bitcoin and Gold are included. This includes an extreme case of how the inclusion of Bitcoin enhances the efficient frontier within the unconstrained framework. The long-only unconstrained portfolios that include Bitcoin, most importantly the optimal portfolio and maximum-return portfolio, outperforms its counterpart portfolios for a given level of risk. The outwards shift and steeper

frontier shows that the Bitcoin's inclusive frontier increases the mean monthly return for a set level of risk. As seen from the figure above, the inclusion of Bitcoin shifts the optimal portfolio up and to the right, therefore taking on more risk, but also resulting in higher returns. In this study, Bitcoin's inclusion increased the optimal efficiency (Sortino ratio) from 0.29 to 1.59 in the optimal portfolio (see Table 6.3). This is an extensive increase per unit of risk. There is a small effect of enhancement on the optimal portfolio when gold is included (0.288 to 0.289), but this is not visible due to the scale of Bitcoin's effect (Table 6.3) (orange frontier is enhanced more than gray frontier but is not visible due to scale of Bitcoin enhancement)

As expected, Bitcoin has a zero weighting, and is excluded, in the minimum-semideviation portfolio. Despite Bitcoin's large diversification effect, it is too volatile and thus is not included in the optimal minimum-risk portfolio. As soon as an investor increases his/her risk tolerance sufficiently, Bitcoin is included in the portfolio, and the mean monthly return increases sharply for a given level of risk. This is in line with Brière et al (2015)'s efficient frontier. The minimum-semideviation portfolios, with and without the allowance of the inclusion of Bitcoin, both have a semideviation and return of 0.24% and 0.54% respectively (due to its zero weighting in both), whilst the inclusion of gold helped decrease risk under the minimum-semideviation portfolio slightly, from 0.242% to 0.240% (see Figure 6.5).

In the portfolio that optimises efficiency (the optimal portfolio including Bitcoin), there is a 12.74% weighting in Bitcoin. The 12.74% allocation to Bitcoin increases the mean monthly return by a factor of four (from 1.05% to 3.97%), with a relatively low increase in semideviation (1.31% to 2.07%) (see Figure 6.5 and Table 6.3). The true return effect of Bitcoin is ultimately shown in the maximum-return portfolio, where mean monthly return is increased from 1.30% to 24.77% (18 times), while the increase in monthly semideviation is relatively less, from 3.28% to 18.53% (4.5 times) (see Appendix A1.9 & A1.10). The above results are consistent with the findings of Carpenter (2016), Eisl et al. (2015) and Brière et al. (2015).

Figure 6.6: The efficient frontier of constrained long-only portfolios



This figure displays the efficient frontier shift that take place upon the inclusion of Bitcoin and gold into optimal portfolios

In Figure 6.6 above, a more proportionate effect from the inclusion of Bitcoin into portfolios is seen. These portfolios are constrained to 5% and 10% respectively, in line with the philosophy of Regulation 28, as was explained in Chapter 5. Figure 6.6 shows that, as with the other portfolios, the higher the weighting in Bitcoin, the greater the portfolios are enhanced for a given level of risk, and the further out the efficient frontier shifts. This means that for a given level of portfolio risk, the returns of the portfolios with Bitcoin far surpass those without. As discussed above, the minimum-semideviation portfolio is not effected by the addition of Bitcoin. Due to the high volatility attached to Bitcoin, Bitcoin would not be added and have zero weighting to the optimal minimum-semideviation portfolio even when the constraint is lifted and it is allowed to be added. Again, the inclusion of gold decreases risk under the minimum-semideviation portfolio, from 0.242% to 0.240% (see Figure 6.6). This finding indicates a small diversification effect of gold in decreasing risk and increasing risk-return efficiency.

Under the long-only constrained framework Bitcoin continues to add value and enhance efficiency, provided that a higher level of risk tolerance than the minimum-semivariance portfolio is adopted by an investor. The optimal portfolio risk adjusted return efficiencies are

increased sharply, even at these lower weightings. With a 10% Bitcoin constraint in the optimal portfolio, the efficiency increases from 0.27 to 1.57 (five times) when compared to the exclusive Bitcoin portfolio (see Table 6.3). Semideviation only marginally increased from 1.31% to 1.67%, whilst the mean monthly return increased from 1.02% to 3.29% (see Figure 6.6). However, for the maximum return portfolios the addition of 10% Bitcoin results in the mean monthly return similarity increasing from 1.07% to 3.47% when compared to the exclusive Bitcoin portfolio (see Figure 6.6). This shows that, at the 10% Bitcoin level, the maximum-return portfolios and the optimal portfolio only exhibits small differences in the enhancement of the return (3.29% to 3.47%). This can be seen in Figure 6.6 by the flat line between the optimal and maximum-return portfolios under the 10% constraint. This is due to the full weighting of Bitcoin already being capped under the optimal portfolio and Regulation 28 compliant capping of the appropriate weightings in all asset classes. Thus, the optimisation for maximum return has little room to work with, as both the optimal and the maximum-return portfolios were fully weighted in Bitcoin and had minimal weighting space to increase in other high return asset classes.

Under the lowest weighting in Bitcoin (the 5% constraint), the increase in the efficiency of the optimal portfolio is from 0.27 to 1.37 (four times) (See Table 6.3). This low weighting brought about an interesting result. It was found that, at the 5% level, the semideviation of the optimal portfolio decreased from 1.31% to 0.92% when including Bitcoin, while the mean monthly return increased from 1.02% to 1.94%, as seen in Figure 6.6. This can be seen by the optimal portfolio not only shifting up, but also shifting leftwards when comparing the “No Bitcoin portfolios” to the “Bitcoin 5%” portfolio. This is different to every other Bitcoin inclusion, which shifted the portfolios upwards and rightwards when compared to its counterparts.

Overall, under all frameworks and at any weight, the inclusion of Bitcoin seems to enhance the efficiency of the optimal portfolio, and shifts the efficient frontier outwards. To benefit from this enhancement, the South African investor has to take on a big amount of risk, but is compensated for this by a large expected return. Thus, the inclusion of Bitcoin into a portfolio will be entirely dependent on the mandate on a fund, and the risk appetite of institutional and retail investors. Furthermore, in this study Bitcoin was found to have a much larger diversification and enhancing effect, even at the 5% level, than gold bullion, even though the latter also displays diversification and enhancing attributes. A key reason for this was that

gold's diversification effect was limited by the big downside risk and minimal return gold experienced over the investment period.

6.2.3 Direct comparison of Bitcoin and gold under different frameworks

In this section, the direct comparison between the addition of Bitcoin and the addition of gold bullion to a well-diversified South African portfolio (the base portfolio) will be discussed. As before, the base portfolio is made up of proxies for domestic listed equity, domestic public bonds, domestic listed property, the domestic money market, foreign bonds and foreign equity, with all proxies being the same as before. In contrast to the previous analysis where Bitcoin was added to a diversified portfolio that included gold bullion, in this section the comparison is between equivalent portfolios holding either Bitcoin or gold bullion, but not both. The purpose of this is to compare Bitcoin directly to gold bullion in terms of diversification and asset allocation effects on the portfolio. The practical long-only constrained portfolio under Regulation 28 will be the portfolio framework used as basis.

Portfolio optimisation

This study has subjected Bitcoin under two constraints in order to draw a direct comparison. All the asset class constraints were constrained in line with Regulation 28, with the exception of Bitcoin, which is not yet formally regulated within Regulation 28. However, in the present study, Bitcoin was constrained to either 10% (the Regulation 28 limit for gold in order to facilitate comparison), or 2.5%, which is probably the most realistic Regulation 28 compliant options, as the latter limits "other assets" to this percentage.

- Constraint 1 is a full Regulation 28 compliant constraint on all asset classes, with no inclusion of Bitcoin or gold (*No Bitcoin and gold*).
- Constraint 2 is a full Regulation 28 compliant constraint on all asset classes, but with gold constrained to 10%, and Bitcoin not included (*Gold 10% and no Bitcoin*).
- Constraint 3 is a full Regulation 28 compliant constraint on all asset classes, but with a 10% constraint on Bitcoin and no inclusion of gold (*Bitcoin 10% and no gold*).
- Constraint 4 is a full Regulation 28 compliant constraint on all asset classes, but with a 2.5% constraint on Bitcoin and no inclusion of gold (*Bitcoin 2.5% and no gold*).

The constraint of 10% on Bitcoin, although probably not Regulation 28 compliant, was used as a first step in order to draw a direct comparison between Bitcoin and gold bullion under the Regulation 28 gold constraint. Similarly, the 2.5% of Bitcoin constraint under the assumption that if Bitcoin was to be included into Regulation 28 today, it would most likely fall under “other assets not referred to in this schedule” which in the regulation is limited to a 2.5% weighting (see section 5.3 under Regulation 28).

Table 6.4: Portfolio optimisation results

Portfolio Optimization Framework	Mean Monthly Optimal Bitcoin Weight	Mean Monthly Semideviation	Mean Monthly Return	Mean Monthly Efficiency (Sortino ration)
Long-Only Constrained (Bitcoin 2.5% and no Gold)	2,50%	0,71%	1,33%	0,9349
Long-Only Constrained (Bitcoin 10% and no Gold)	10,00%	1,64%	3,25%	1,5685
Long-Only Constrained (Gold 10% and no Bitcoin)	0,00%	1,31%	1,02%	0,2671
Long-Only Constrained (No Bitcoin or Gold)	0,00%	1,35%	1,03%	0,2645

This table displays the results of each optimising portfolio under the long-only constrained portfolio framework with different inclusions and asset weighting caps. All data is presented as monthly means over the investment period namely, July 2010 to February 2018.

Key

- *Bitcoin 2.5% and no gold-* Bitcoin constrained to 2.5%, and gold not included in the optimisation
- *Bitcoin 10% and no gold-* Bitcoin constrained to 10%, and gold not included in the optimisation
- *Gold 10% and no Bitcoin –* Gold is constrained to 10% and Bitcoin not included in the optimisation
- *No Bitcoin and gold-* The base portfolio with neither Bitcoin nor old included

From Table 6.4 above it can be seen that the effect of Bitcoin’s inclusion into a standard South African portfolio (without gold) has a much greater risk-return efficiency enhancing effect across all constraints.

10% constraint comparison

Under a direct comparison on weight, namely with both gold and Bitcoin at a 10% constraint, it can be seen that the Bitcoin inclusion helps enhance the risk-return efficiency far more than the gold bullion addition. The base portfolio (no Bitcoin or gold) has a mean monthly semideviation and return of 1.35% and 1.03%, respectively. When gold bullion is included at a 10% constraint the mean monthly semideviation actually decreases from 1.35% to 1.31%, while the mean monthly return decreases by a lesser proportion (1.03% to 1.02%). This lesser than proportionate decrease in risk to return shows that the inclusion of gold bullion slightly improves the portfolio's risk-return profile. However, when Bitcoin is added at the 10% constraint the mean semivariance increased from the base of 1.35% to 1.64%. This is understandable due to Bitcoin's large risk attached to its returns (see Table 6.1). On the other hand, it can be seen that the base mean monthly return increases fundamentally from 1.03% to 3.35% (*i.e.* a more than three times increase). These results are in line with the findings of Eisl et al. (2015), Brière et al. (2015), and Carpenter (2016).

This can be seen in the fact that the base portfolio (no Bitcoin or gold) shows an efficiency (Sortino ratio) of 0.2645, and that upon the inclusion of gold the efficiency increases only slightly, to 0.2671. When Bitcoin is added under a 10% constraint, the efficiency increases largely from the 0.2645 of the base portfolio to 1.5685 (a near six times increase). This huge increase is driven by Bitcoin's excessive returns as weight changes helped to elevate the risk attached to those returns.

It must be noted that under the 10% constraint, gold never reached the maximum level as Bitcoin did at the 10% level. Thus, gold's maximum weighting under the '*Gold 10% and no Bitcoin*' portfolio was only 3.47%, whereas it can be seen in Table 6.3 that the optimum Bitcoin holding always seems to be at the limit set by the chosen constraint within this study. This causes asset allocation shifts as discussed in Section 6.1 of this study. The main effect is that, upon the inclusion of Bitcoin, the asset allocation shifts away from local equities, local property and foreign equities, and towards Bitcoin, local bonds and local cash. As explained before, local bonds and local cash have the two strongest negative correlations to Bitcoin and Foreign equity, and the lowest semideviations (2.19% and 0.26%, respectively).

10% Gold constraint vs 2.5% Bitcoin constraint

Under current circumstances it would be prudent to assume that Bitcoin would be included under Regulation 28 at a 2.5% weighting cap under the category of 'other assets not referred to in this schedule'. From the table above, the base portfolio (no Bitcoin or gold) has a mean

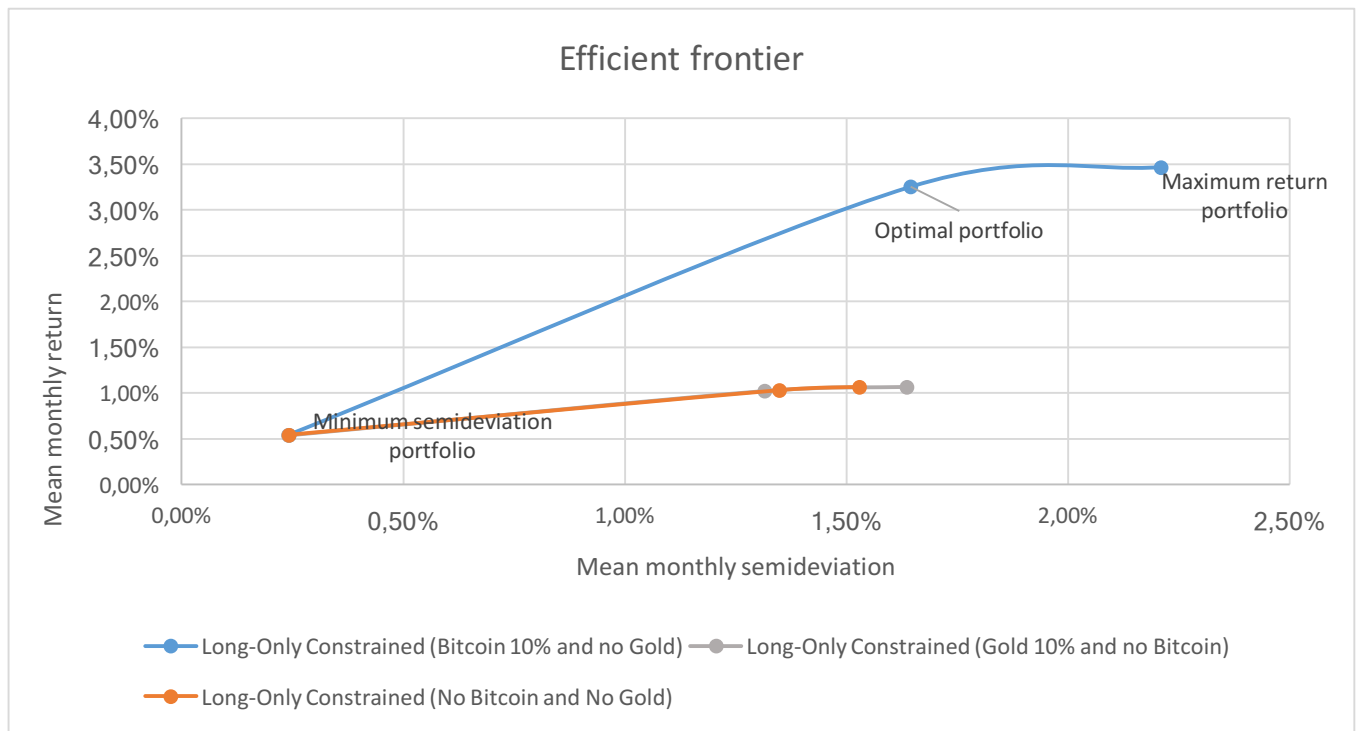
monthly semideviation and return of 1.35% and 1.03%, respectively. As stated in table 6.4 , when gold bullion is included at a 10% constraint, the mean monthly semideviation decreases from 1.35% to 1.31%, while the mean monthly return decreases by a lesser proportion (1.03% to 1.02%), thus enhancing the risk-return profile of the portfolio only slightly, from 0.2645 to 0.2671.

When Bitcoin is added at the 2.5% constraint, the mean semivariance actually decreases from the base of 1.35% to 0.71% , a greater decrease than that caused by gold bullion's addition. It can also be seen that the base mean monthly return increases from 1.03% to 1.33%, thus enhancing the risk-return profile of the portfolio, from 0.264 to 0.934. These results mimic the results from the above (table 6.3) analysis with Bitcoin at the 5% level, and show that Bitcoin weightings lower than 5% seem to reduce portfolio risk. These findings differ from the findings of Eisl et al. (2015), Brière et al. (2015) and Carpenter (2016) in the US context. This difference is driven by South Africa's asset classes different correlations and risk profiles to those of the US, and in particular the low to negative asset correlations with Bitcoin.

In the present study, the inclusion of Bitcoin at levels of 5% or lower (*e.g.* 2.5%) seemed to decrease risk while increasing the mean return. Thus, the 2.5% constraint above showed the same features as the 5% constraint discussed earlier in this study. At these levels of Bitcoin, it can be seen that the asset allocation shift previously discussed is exacerbated. The asset allocation again shifts away from local equities, local property and foreign equities and goes into Bitcoin, local bonds and local cash. As explained above, local bonds and local cash have the two strongest negative correlations to Bitcoin and foreign equity, as well as the two lowest semideviations (2.19% and 0.26% respectively). Thus, the high risk attached to Bitcoin and foreign equity is diversified by the equivalent increase in the local bonds and local cash holdings.

Efficient frontiers

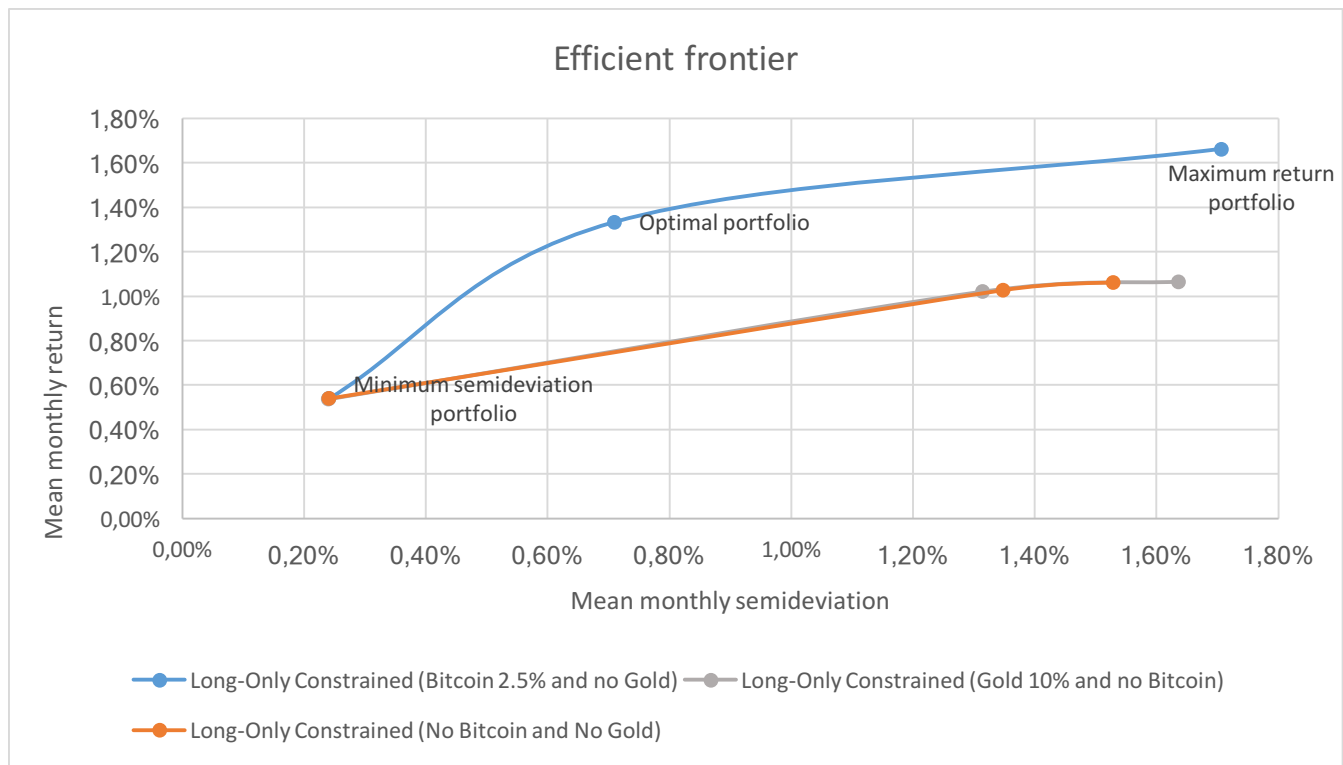
Figure 6.8: The efficient frontier of constrained long-only portfolios



This figure shows the efficient frontier shift that takes place upon the inclusion of Bitcoin and gold, at the 10% constraint, into optimal portfolios.

Figure 6.8 shows the efficient frontier shifts that occur upon the addition of Bitcoin under a 10% constraint *versus* adding gold under a 10% constraint to the base portfolio. As can be seen in figure 6.8, the addition of Bitcoin shifts the efficient frontier out and drastically enhances the efficient frontier, resulting in an outwards shift and steeper frontier. This implies that Bitcoin's inclusive frontier increases the mean monthly return for a set level of risk. Bitcoin's inclusion increased the optimal efficiency (Sortino ratio) from 0.2645 to 1.59 (see Table 6.3) in the optimal portfolio. This is an extensive increase per unit of risk. There is a small enhancement effect on the optimal portfolio when gold is included (0.2645 to 0.2671). However, gold is included in the minimum-semivariance portfolio as it helps reduce risk. As stated before, Bitcoin is not included in the minimum-semivariance portfolio due to the high levels of risk attached to it, but Bitcoin starts to add value and enhance efficiency as the investor starts to take on more risk than the minimum-semivariance portfolio.

Figure 6.9: The efficient frontier of constrained long-only portfolios



This figure shows the efficient frontier shift that take place upon the inclusion of Bitcoin at the 2.5% and gold at the 10% constraint, into optimal portfolios.

Figure 6.9 shows that the efficient frontier shifts upon adding Bitcoin at a 2.5% constraint, *versus* adding gold at 10% constraint, to the base portfolio. The figure shows that at the 2.5% level, Bitcoin does not enhance the efficient frontier as much as it does at the 10% level, but still does enhance the frontier. It can further be seen that there is an outwards shift and steeper frontier when Bitcoin is singularly added. This means that, the Bitcoin inclusive frontier will increase the mean monthly return for a set level of risk. The inclusion of Bitcoin at the 2.5% level shifted the optimal portfolio up and to the left, therefore taking on less risk, but still producing higher returns. The reduction in semideviation in the optimal portfolio as explained above is driven by asset allocation changes to help diversify out risk. The addition of gold bullion to the base portfolio also shifts the optimal portfolio to the left, indicating that it too decreases the semideviation with only a slight drop in mean return, thus also increasing the risk-return characteristics. Gold bullion was also included in the minimum-semideviation portfolio, as it helps reduce risk even at the minimal optimisation level.

Overall, it can be seen that Bitcoin, when compared directly, is a better portfolio diversifier to a base/standard South African portfolio than gold bullion.

Chapter 7: Conclusion

This section outlines the concluding remarks and findings, and also sets out limitations to this study and suggested avenues for further research.

7.1 Review of study

Bitcoin is a digital, unregulated and partially anonymous currency, which is not backed by any government or legal entity. There exists very limited academic research on cryptocurrencies as diversification assets within a portfolio context. All previous academic research on the portfolio diversification abilities of cryptocurrencies was done within developed market contexts (mainly the United States¹⁴), and not in an emerging market context such as South Africa. Therefore, this study seeks to investigate the portfolio diversification and optimisation abilities and asset allocation effects of the inclusion of Bitcoin in a portfolio from the perspective of a South Africa investor, using August 2010 – February 2018 as the sample period. This is both a much longer period (91 months) compared to previous studies, but also includes a bear trend towards the end of the period. The study further critically compares Bitcoin to gold bullion in terms of the abilities of each asset as a diversifier and portfolio optimiser.

This study set out to answer the following research questions:

- Does Bitcoin enhance the efficient frontier and, in turn, the risk-return profile of a well-diversified South African portfolio?
- Does Bitcoin affect the asset allocation of a well-diversified South African portfolio?
- Does Bitcoin have better diversification abilities than gold bullion within a South African portfolio?

Firstly, the study assessed the correlation between Bitcoin and more traditional asset classes. It was found that, in accordance with international literature, Bitcoin's correlation with most asset classes typically found in South African portfolios are low. The risk-return measure of an optimal portfolio indicates to investors the risk adjusted return of the portfolio, and is therefore a comparable measure when assessing relative performance of optimal portfolios.

¹⁴ See, for example, Baur, Lee and Hong (2015); Yermack, (2013); Burniske and White, (2016); Wu and Pandey (2014); Brière, Oosterlinck and Szafarz (2015); and Eisl, Gasser and Weinmayer (2015)

Bitcoin returns are not normally distributed, which renders the commonly known Sharpe ratio unsuitable. A mean-semivariance framework allows for Bitcoin's non-normal distributed returns to be interpreted by only assessing the downside risk.

The study found that Bitcoin enhances the risk-return efficiency of a South African portfolio, and displays characteristics of a portfolio diversifier. Firstly, Bitcoin enhances the risk-return efficiency (Sortino ratio) and shifts the efficient frontier up and outwards when Bitcoin is added to portfolios containing gold bullion under every portfolio framework. It was also discovered that Bitcoin, at weighting levels of 5% or lower, not only improves the risk-return profile, but further reduces the outright risk of the optimal portfolio.

The addition of Bitcoin also displayed an asset allocation shift from higher risk assets such as the All share index, the SA Property index and the MSCI world index to allow for the lower risk and higher negatively correlated assets; namely SA all bond index and SA STeFI cash index. The higher weightings in the portfolio helped elevates Bitcoins risk and enhance the portfolio risk-return profile.

Bitcoin was found to be superior to gold in enhancing the risk-return efficiency (Sortino ratio) and shifting the efficient frontier up and outwards, including when directly compared under the constraints of Regulation 28 (a 2.5% weighting cap for Bitcoin, and 10% for gold). At a 2.5% weighting Bitcoin reduced the portfolio's overall risk and proved not only to add return enhancements greater than the proportionate risk increase, but to also reduce the risk of the portfolio.

Overall, Bitcoin appears to be a new option as a portfolio diversifier for a South African investor. The addition of Bitcoin to a portfolio does, however, drastically increase the risk attached to the portfolio, and it is therefore only appropriate for risk tolerant investors. In general the addition of Bitcoin dramatically enhances the risk return trade-off. However, at lower weights of Bitcoin (<5%) the study finds an actual reduction in portfolio risk, while still increasing the portfolio return.

7.2 Limitations of study

A limitation to this study is the limited sample period of August 2010 to February 2018 (91 months), specifically as correlations can change vastly under stress, and only capturing the correlations in a bull market could skew the results of such correlations (Brière et al. 2015).

The data used covers an extreme bull run and only includes a quite short period of the bear trend that Bitcoin has been in since December 2017 (Appendix A1.4). The limitation is fundamentally that cryptocurrencies (including Bitcoin) are a new asset class, and as of yet does not have the reasonably stable and sustainable history that is ideally required for diversification testing. True robust conclusions or backing of the conclusions made in this study can only be made once more price data and further cycle trends of Bitcoin's price data are available.

A further limitation is transaction costs, which were not taken into account in this study. Buying, selling and transferring Bitcoin has been seen to be quite costly in the past, and increased to all-time highs of \$55 per transaction over December 2017 as the amount of transactions caused the block size (1MB) to be capped. These fees have since decreased back down to as low as \$0.8 per transaction in July 2018 (Bitinfochart, 2018). The need for bigger blocks will, however, drive transaction fees down over time, thus bringing reality in line with the findings of this study.

7.3 Further research

Firstly, this study should be repeated once more robust price data is available – in other words, data that is fully sustainable and includes bull, bear and sideways Bitcoin markets. Secondly, the same analysis can be conducted on different cryptocurrencies, or compare the diversification effects of different cryptocurrencies in a standard South African portfolio (or any other portfolio). Further research may look at changing the data points of the data to daily, weekly or even biweekly in order to obtain more data points, but this will run the risk of picking up 'noise' in the data. The research could also change the methodology to implement a mean-CVaR methodology in the same context of a South African investor, to see how the different methodologies will affect results.

References

- Acheson, N. 2018. How can I buy bitcoins? - CoinDesk. Available: <https://www.coindesk.com/information/how-can-i-buy-bitcoins/> [2018, July 12].
- Amadeo, K. 2017. 5 Traits of Emerging Market Countries. Available: <https://www.thebalance.com/what-are-emerging-markets-3305927> [2017, September 04].
- Asian Development Bank. 2017. Bonds turnover ratio. Available: https://asianbondsonline.adb.org/regional/data/bondmarket.php?code=Bond_turn_ratio [2017, July 21].
- Baur, D. & Lucey, B. 2010. Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. *Financial Review*. 45(2):217-229. DOI: 10.1111/j.1540-6288.2010.00244.x.
- Baur, D. & McDermott, T. 2010. Is gold a safe haven? International evidence. *Journal of Banking & Finance*. 34(8):1886-1898. DOI: 10.1016/j.jbankfin.2009.12.008.
- Baur, D., Lee, A. & Hong, K. 2015. Bitcoin: Currency or Investment? *SSRN Electronic Journal*. DOI: 10.2139/ssrn.2561183.
- Beer, J. 2017. *Why bitcoin prices are rising -- way more than a bit*. Available: <https://www.cbsnews.com/news/bitcoin-prices-rising-double-in-value-cryptocurrency/> [2017, September 01].
- Bitcoin currency statistics. 2018. Available: <https://blockchain.info/stats> [2018, July 12].
- Bitinfochart. 2018. Bitcoin Avg. Transaction Fee chart. Available: <https://bitinfocharts.com/comparison/bitcoin-transactionfees.html> [2018, July 12].
- Blockchain charts. 2018. Available: <https://blockchain.info/charts> [2018, July 12].
- Brière, M., Oosterlinck, K. & Szafarz, A. 2015. Virtual currency, tangible return: Portfolio diversification with bitcoin. *Journal of Asset Management*. 16(6):365-373. DOI: 10.1057/jam.2015.5.

- Burniske, C. & White, A. 2016. *Bitcoin: Ringing the bell for a new asset class*. (White paper) Ark invest research. Available: https://research.ark-invest.com/hubfs/1_Download_Files_ARK-Invest/White_Papers/Bitcoin-Ringing-The-Bell-For-A-New-Asset-Class.pdf [2017, July 19].
- Carpenter, A. 2016. Portfolio diversification with Bitcoin. *Journal of undergraduate research in finance*. Available: <http://jurf.org/wp-content/uploads/2017/01/carpenter-andrew-2016.pdf> [2017, July 19].
- Chua, J., Sick, G. & Woodward, R. 1990. Diversifying with Gold Stocks. *Financial Analysts Journal*. 46(4):76-79. DOI: 10.2469/faj.v46.n4.76.
- Courtney Capital. 2017. JSE Top 40 shares - Courtney Capital. Available: <http://www.courtneycapital.co.za/jse-top-40-shares/> [2017, September 04].
- CryptoCurrency Market Capitalisations. | CoinMarketCap 2018. Available: <https://coinmarketcap.com/currencies/> [2018, July 12].
- de Groot, W. & Zhou, W. 2015. Strategic allocation to quantitative emerging markets strategies. (White Paper). *Rebecon investment engineers*.
- DeMiguel, V., Garlappi, L. & Uppal, R. 2007. Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?. *Review of Financial Studies*. 22(5):1915-1953. DOI: 10.1093/rfs/hhm075.
- Department of the National Treasury. 2017. *Debt Management Report 2015/16*. Pretoria: Department of the National Treasury
- Doguet, J 2012. The Nature of the Form: Legal and Regulatory Issues Surrounding the Bitcoin Digital Currency System. *Louisiana Law Review*. 1120-1153.
- Dyhrberg, A. 2016. Hedging capabilities of bitcoin. Is it the virtual gold?. *Finance Research Letters*. 16:139-144. DOI: 10.1016/j.frl.2015.10.025\.
- Eisl, A., Gasser, S. & Weinmayer, K. 2015. Caveat Emptor: Does Bitcoin Improve Portfolio Diversification?. *SSRN Electronic Journal*. DOI: 10.2139/ssrn.2408997.

- Estrada, J. 2007. Mean-semivariance optimisation: A heuristic approach. *Journal of Applied Finance*. 18(1): 57-72.
- European Central Bank (ECB). 2012. *Virtual Currency Schemes*. Available: <https://www.ecb.europa.eu/pub/pdf/other/virtualcurrencyschemes201210en.pdf> [3 July 2017].
- Fama, E. F. 1970. Efficient Capital Market: A Review of Theory and Emperical Work. *American Finance Association*. 25(2): 383-417.
- Fishburn, P.C. 1977. Mean-Risk Analysis with Risk Associated with Below-Target Returns, *The American Economic Review*, 67 (2), pp. 116-126.
- Grinberg, R. 2012 Bitcoin: An innovative alternative digital currency. *Hastings Science & Technology Law Journal* 4: 160–207.
- Hammoudeh, S. & Yuan, Y. 2008. Metal volatility in presence of oil and interest rate shocks. *Energy Economics*. 30(2):606-620. DOI: 10.1016/j.eneco.2007.09.004.
- Hassan, S. 2013. *South African Capital Markets: An Overview*. Working Paper Series WP/13/04. (White Paper). South African Reserve Bank.
- Hassan, S. & Van Biljon, A. 2010. The Equity Premium and Risk-Free Rate Puzzles in a Turbulent Economy: Evidence from 105 Years of Data from South Africa. *South African Journal of Economics*. 78(1):23-39. DOI: 10.1111/j.1813-6982.2010.01237.x.
- Jaffe, J. 1989. Gold and Gold Stocks as Investments for Institutional Portfolios. *Financial Analysts Journal*. 45(2):53-59. DOI: 10.2469/faj.v45.n2.53.
- Johannesburg Stock Exchange. 2013. *Debt Market*. Available: <https://www.jse.co.za/trade/debt-market> [3 July 2017].
- Johannesburg Stock Exchange. 2013. *Index series*. Available: <https://www.jse.co.za/services/market-data/indices/ftse-jse-africa-index-series/headline> [3 July 2017].

- Leland, H E. 1999. Beyond Mean-Variance: Performance Measurement in a Nonsymmetrical World. *Financial Analyst Journal*. January/February: 29-36.
- McNeil, A., Frey, R., and Embrechts, P. (2005). Quantitative Risk Management. *Princeton University Press*.
- Mangani, R. 2007. Distributional properties of JSE prices and returns. *Investment Analysts Journal*. 66: 57-72.
- Markowitz, H. 1952. Portfolio Selection. *The Journal of Finance*. 7(1):77-91
- Markowitz, H M. 1959. Portfolio Selection: efficient diversification of investments. New York: John Wiley & Sons Inc.
- Michie, R. 1999 The London Stock Exchange. A History, Oxford: *Oxford University Press*.
- MSCI. 2017. MSCI Emerging Market Index. Retrieved from MSCI:
<https://www.msci.com/documents/10199/24d5baf3-d8ad-4280-adbf-f727a9cfa4b4>
- Nakamoto, S. 2008. *Bitcoin: A peer-to-peer electronic cash system*.
- Rockafellar, R. & Uryasev, S. 2000. Optimisation of conditional value-at-risk. *The Journal of Risk*. 2(3):21-41. DOI: 10.21314/jor.2000.038.
- Salomons, R. & Grootveld, H. 2003. The equity risk premium: emerging vs. developed markets. *Emerging Markets Review*. 4(2):121-144. DOI: 10.1016/s1566-0141(03)00024-4.
- Serhan, A. 2014. Developed versus emerging markets - Morningstar.com.au. Available: <http://www.morningstar.com.au/specialreports/funds/topicweek-developed-emerging/6729> [2017, September 04].
- StockMarketClock. 2017,. Johannesburg Stock Exchange. Available: <https://www.stockmarketclock.com/exchanges/jse#> [2017, September 04].

The World Bank. 2017. An analysis of trends in cost of remittance services. Retrieved July 2017, 3, from The World Bank Group:

https://remittanceprices.worldbank.org/sites/default/files/rpw_report_june_2017.pdf

Thompson, C. 2017. *How does the Blockchain Work? (Part 1) – The Blockchain Review – Medium*. Available: <https://medium.com/blockchain-review/how-does-the-blockchain-work-for-dummies-explained-simply-9f94d386e093> [2017, September 01].

Tobin J. 1958. Liquidity Preference as Behavior Towards Risk. *The Review of Economic Studies*. 25(2): 65-86.

Turpin, J. 2014. Bitcoin: The Economic Case for a Global, Virtual Currency Operating in an Unexplored Legal Framework. *Indiana Journal of Global Legal Studies*. 21(1):335-368. DOI: 10.2979/indjglolegstu.21.1.335.

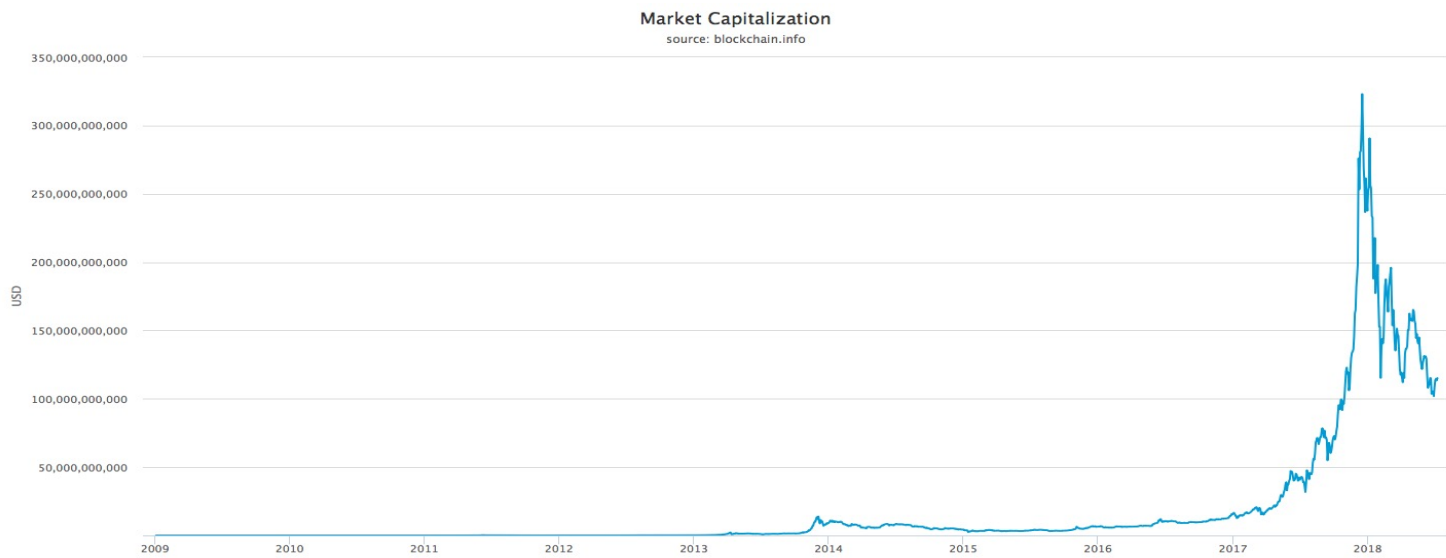
Wu, C. & Pandey, V. 2014. The Value of Bitcoin in Enhancing the Efficiency of an Investor's Portfolio. *Journal of Financial Planning*, 44-52. Available: https://www.researchgate.net/profile/Vivek_Pandey30/publication/280883667_The_value_of_Bitcoin_in_enhancing_the_efficiency_of_an_investor's_portfolio/links/55ca6d5b08aea2d9bdcc00d4.pdf [2018, July 12].

Yermack, D. 2013. Is Bitcoin a Real Currency? *SSRN Electronic Journal*. DOI: 10.2139/ssrn.2361599.

Appendices

Appendix A1.1 – Bitcoin's market capitalization

Displays the total value of Bitcoins supply that is currently in circulation. This is calculated through the daily average market price across major exchanges.

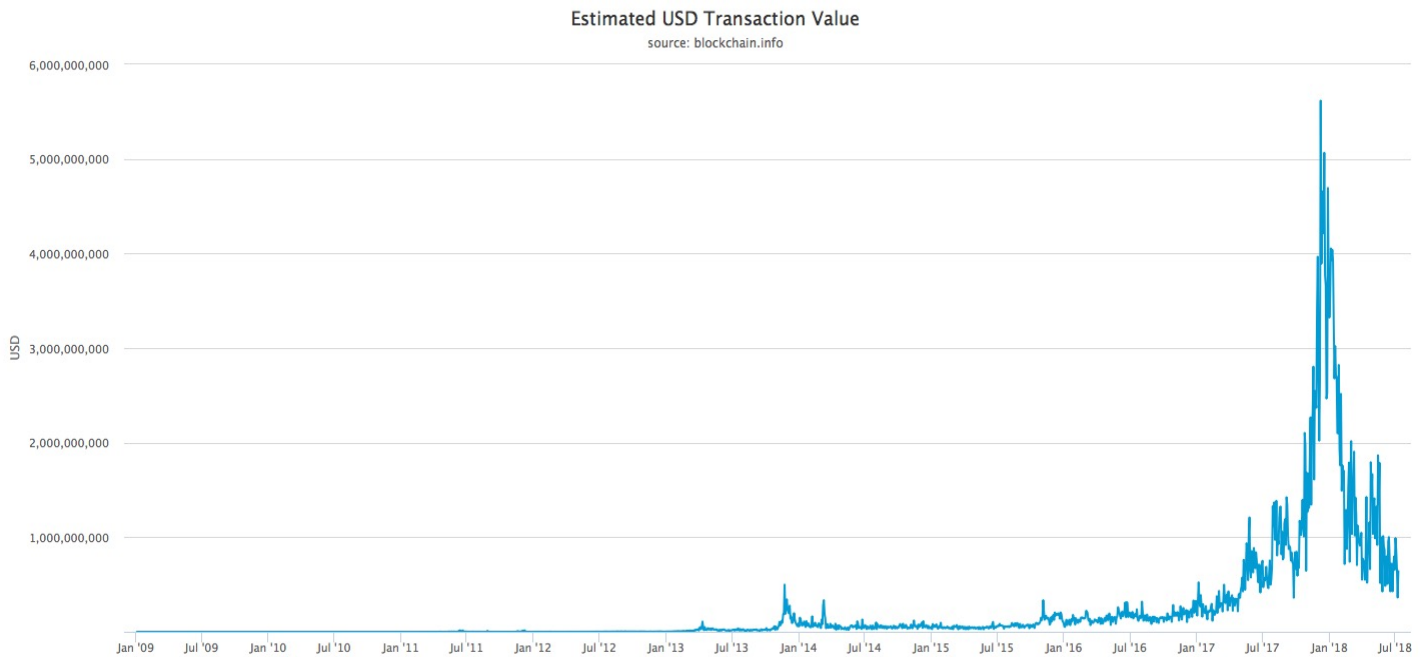


Source: (Blockchain charts, 2018)

Blockchain.info (<https://blockchain.info/charts/market-cap>)

Appendix A1.2 – Bitcoins estimated Transaction value (in USD)

Displays the total estimated value of the transactions on the Bitcoin blockchain. This appendix does not include Bitcoin's returned to sender as change.

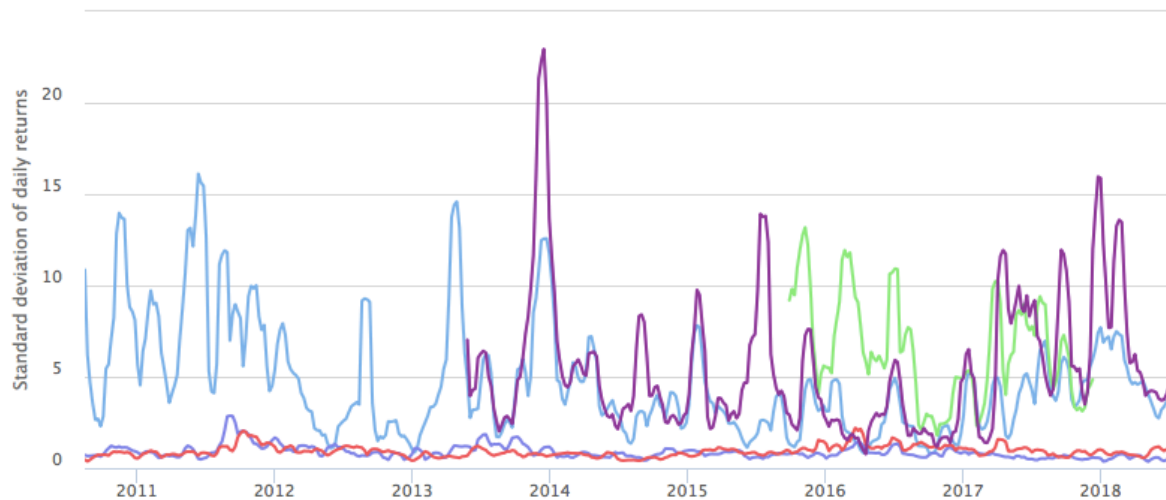


Source: (Blockchain charts, 2018)

Blockchain.info(<https://blockchain.info/charts/estimated-transaction-volume?daysAverageString=7×pan=all>)

Appendix A1.3 - A Bitcoin volatility comparison

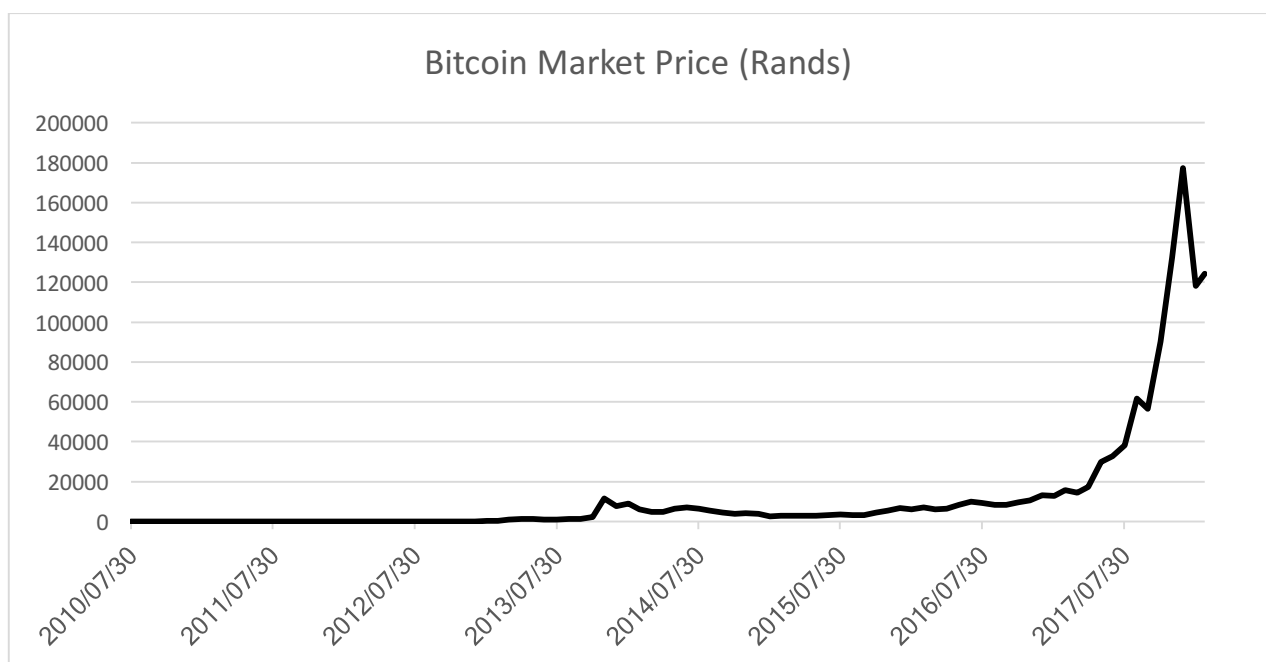
Displays and compares Bitcoin's 30-day volatility (Blue) to the 30-day volatility of Ethereum (ETH) (Purple), Litecoin (LTC)(Green), USDZAR (Red) and gold bullion (Dark Blue).



source: <https://www.buybitcoinworldwide.com/volatility-index/>

Appendix A1.4 – Market price of Bitcoin (Rands)

Displays the historical monthly close market price of Bitcoin denominated in rands. USD Bitcoin monthly close price was converted to rands at the corresponding monthly close price of the USDZAR exchange rate cross.



Appendix A1.5 – South Africa vs USA inflation index

Displays the yearly inflation rate of South Africa and the USA over the last 56 years.

	SACPIYOY INDEX	CPI YOY Index (US)
Date	Last PX	Last PX
05/31/2017	5,4	1,9
12/30/2016	6,7	2,1
12/31/2015	5,3	0,7
12/31/2014	5,3	0,8
12/31/2013	5,4	1,5
12/31/2012	5,7	1,7
12/30/2011	6,2	3
12/31/2010	3,5	1,5
12/31/2009	6,3	2,7
12/31/2008	9,6	0,1
12/31/2007	8,8	4,1
12/29/2006	5,8	2,5
12/30/2005	3,6	3,4
12/31/2004	3,5	3,3
12/31/2003	0,2	1,9
12/31/2002	12,4	2,4
12/31/2001	4,7	1,6
12/29/2000	6,8	3,4
12/31/1999	2,1	2,7
12/31/1998	9,1	1,6
12/31/1997	6,2	1,7
12/31/1996	9,5	3,3
12/29/1995	6,9	2,5
12/30/1994	9,6	2,7
12/31/1993	9,6	2,7
12/31/1992	9,6	2,9
12/31/1991	16,1	3,1
12/31/1990	14,6	6,1

12/29/1989	15,4	4,6
12/30/1988	12,4	4,4
12/31/1987	15,2	4,4
12/31/1986	18	1,1
12/31/1985	18,7	3,8
12/31/1984	11,9	3,9
12/30/1983	11,7	3,8
12/31/1982	13,2	3,8
12/31/1981	12,8	8,9
12/31/1980	17,5	12,5
12/31/1979	17,6	13,3
12/29/1978	9,7	9
12/30/1977	10,7	6,7
12/31/1976	7,7	4,9
12/31/1975	18,2	6,9
12/31/1974	10	12,3
12/31/1973	11,1	8,7
12/29/1972	5,9	3,4
12/31/1971	6,3	3,3
12/31/1970	6,7	5,6
12/31/1969	7,1	6,2
12/31/1968	0	4,7
12/29/1967	0	3
12/30/1966	7,7	3,5
12/31/1965	0	1,9
12/31/1964	8,3	1
12/31/1963	0	1,6
12/31/1962	0	1,3
12/29/1961	0	0,7

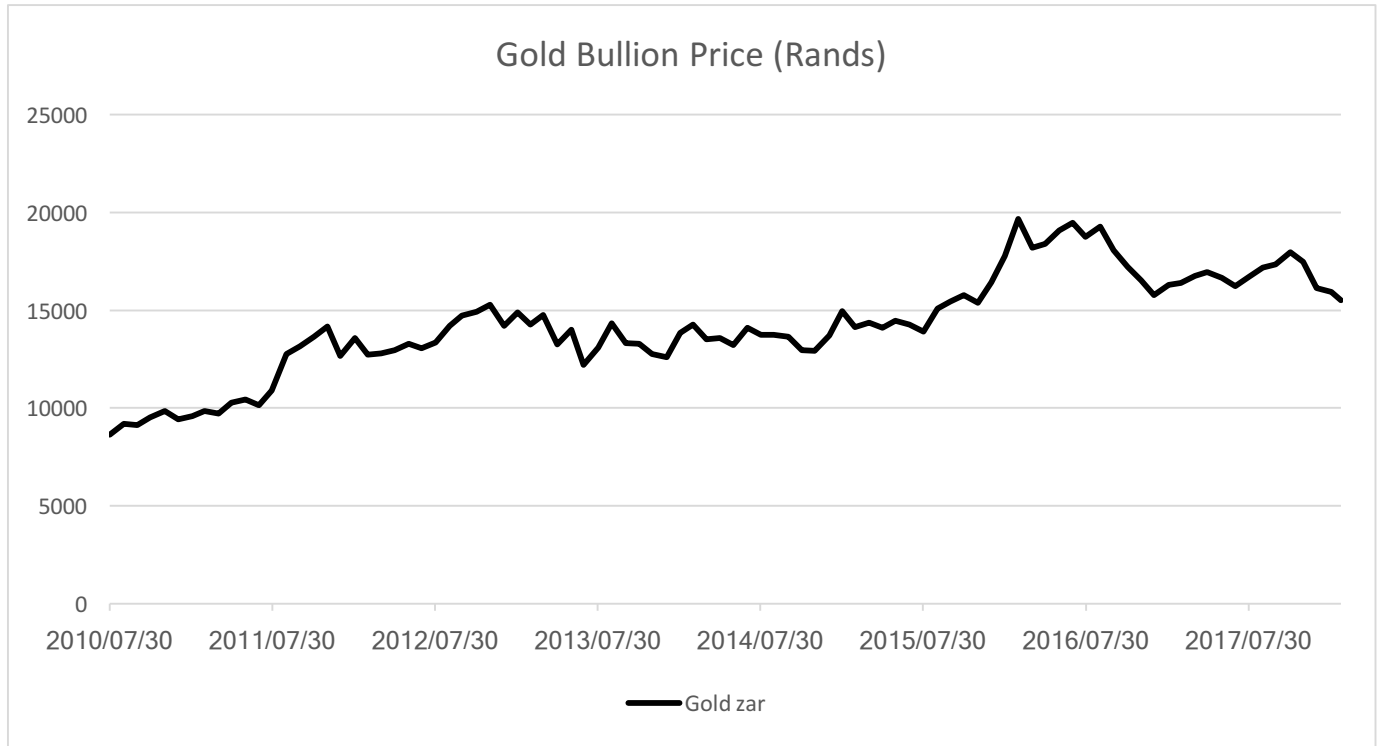
Appendix A1.6 – South Africa vs USA cash investment

Displays the year end cash interest rate of the South African 3-month JIBAR and the USA 3-month LIBOR.

	JIBA3M	US0003M
	Index	Index
Date	Last PX	Last PX
2017/07/07	7,342	1,30411
12/30/2016	7,358	0,99789
12/31/2015	6,625	0,6127
12/31/2014	6,125	0,2556
12/31/2013	5,217	0,2461
12/31/2012	5,125	0,306
12/30/2011	5,595	0,581
12/31/2010	5,55	0,30281
12/31/2009	7,229	0,25063
12/31/2008	11,425	1,425
12/31/2007	11,25	4,7025
12/29/2006	9,176	5,36
12/30/2005	7,05	4,53625
12/31/2004	7,471	2,56438
12/31/2003	7,725	1,15188
12/31/2002	13,485	1,38
12/31/2001	9,782	1,88125
12/29/2000	10,531	6,39875
12/31/1999	11,239	6,00125

Appendix A1.7 - Market price of Gold (Rands)

Displays the historical monthly close market price of Gold Bullion denominated in rands. USD Gold Bullion monthly close price was converted to rands at the corresponding monthly close price of the USDZAR exchange rate cross.



Appendix A1.8 – Long-Only unconstrained optimally efficient portfolios (including Bitcoin)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Unconstrained (Bitcoin)	Semideviation	Return
Maximum-Return portfolio	18,5306%	24,7684%
Optimal portfolio	2,0737%	3,9717%
Minimum-Semideviation portfolio	0,2400%	0,5368%

Appendix A1.9 – Long-Only unconstrained optimally efficient portfolios (excluding Bitcoin)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Unconstrained (No Bitcoin)	Semideviation	Return
Maximum-Return portfolio	3,2775%	1,2989%
Optimal portfolio	1,3106%	1,0489%
Minimum-Semideviation portfolio	0,2400%	0,5368%

Appendix A1.10 – Long-Only unconstrained optimally efficient portfolios (excluding Bitcoin and Gold)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Unconstrained (No Bitcoin or Gold)	Semideviation	Return
Maximum-Return portfolio	3,2775%	1,2989%
Optimal portfolio	1,2961%	1,0436%
Minimum-Semideviation portfolio	0,2415%	0,5399%

Appendix A1.11 – Long-Only Constrained optimally efficient portfolios (including 5% Bitcoin)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Constrained (Bitcoin 5%)	Semideviation	Return
Maximum-Return portfolio	2,1694%	2,2714%
Optimal portfolio	0,9241%	1,9357%
Minimum-Semideviation portfolio	0,2400%	0,5368%

Appendix A1.12 – Long-Only Constrained optimally efficient portfolios (including 10% Bitcoin)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Constrained (Bitcoin 10%)	Semideviation	Return
Maximum-Return portfolio	2,5135%	3,4729%
Optimal portfolio	1,6674%	3,2902%
Minimum-Semideviation portfolio	0,2400%	0,5368%

Appendix A1.13 – Long-Only Constrained optimally efficient portfolios (excluding Bitcoin)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Constrained (No Bitcoin)	Semideviation	Return
Maximum-Return portfolio	1,8851%	1,0699%
Optimal portfolio	1,3148%	1,0216%
Minimum-Semideviation portfolio	0,2400%	0,5368%

Appendix A1.14 – Long-Only Constrained optimally efficient portfolios (excluding Bitcoin and Gold)

This table shows the mean monthly semideviations and returns for the three optimized portfolios marked on the efficient frontier. Namely the maximum-return, optimal and Minimum semideviation portfolio

Long-Only Constrained (No Bitcoin or Gold)	Semideviation	Return
Maximum-Return portfolio	2,0552%	1,0656%
Optimal portfolio	1,3484%	1,0271%
Minimum-Semideviation portfolio	0,2415%	0,5399%

Appendix A1.14 – Regulation 28

		Column 1	Column 2	
			Limits being the maximum percentage of aggregate fair value of total assets of fund	
Item		Categories of assets	Per issuer/entity, as applicable	For all issuers/entities
1.	CASH			100%
1.1	Notes and coins; any balance or deposit in an account held with a South African bank; A money market instrument issued by a South African bank including an Islamic liquidity management financial instrument; Any positive net balance in a margin account with an exchange; and Any positive net balance in a settlement account with an exchange, operated for the buying and selling of assets.		25%	100%
1.2	Any balance or deposit held with a foreign bank; A money market instrument issued by a foreign bank including an Islamic liquidity management financial instrument;		5%	
2.	DEBT INSTRUMENTS INCLUDING ISLAMIC DEBT INSTRUMENTS			100% for debt instruments issued by or guaranteed by the Republic, otherwise 75%
2.1	Inside the Republic and foreign assets			
	(a)	Debt instruments issued by, and loans to, the government of the Republic, and any debt or loan guaranteed by the Republic		100%
	(b)	Debt instruments issued or guaranteed by the government of a foreign country	10%	
	(c)	Debt instruments issued or guaranteed by a South African bank against its balance sheet: -		75%
	(i)	listed on an exchange with an issuer market capitalisation of R20 billion or more, or an amount or conditions as prescribed	25%	

		(ii)	listed on an exchange with an issuer market capitalisation of between R2 billion and R20 billion, or an amount or conditions as prescribed	15%	
		(iii)	listed on an exchange with an issuer market capitalisation of less than R2 billion, or an amount or conditions as prescribed	10%	
		(iv)	not listed on an exchange	5%	25%
	(d)	Debt instruments issued or guaranteed by an entity that has equity listed on an exchange, or debt instruments issued or guaranteed by a public entity under the Public Finance Management Act, 1999 (Act No. 1 of 1999) as prescribed: -		10%	50%
		(i)	listed on an exchange	10%	50%
		(ii)	not listed on an exchange	5%	25%
	(e)	Other debt instruments: -		5%	25%
		(i)	listed on an exchange	5%	25%
		(ii)	not listed on an exchange	5%	15%
3.	EQUITIES				75%
3.1	Inside the Republic and foreign assets				
	(a)	Preference and ordinary shares in companies, excluding shares in property companies, listed on an exchange: -			75%
		(i)	issuer market capitalisation of R20 billion or more, or an amount or conditions as prescribed	15%	
		(ii)	issuer market capitalisation of between R2 billion and R20 billion, or an amount or conditions as prescribed	10%	
		(iii)	issuer market capitalisation of less than R2 billion, or an amount or conditions as prescribed	5%	
	(b)	Preference and ordinary shares in companies, excluding shares in property companies, not listed on an exchange		2.5%	10%

4.	IMMOVABLE PROPERTY			25%
4.1	Inside the Republic and foreign assets			
	(a)	Preference shares, ordinary shares and linked units comprising shares linked to debentures in property companies, or units in a Collective Investment Scheme in Property, listed on an exchange:-		25%
	(i)	issuer market capitalisation of R10 billion or more, or an amount or conditions as prescribed	15%	
	(ii)	issuer market capitalisation of between R3 billion and R10 billion, or an amount or conditions as prescribed	10%	
	(iii)	issuer market capitalisation of less than R3 billion, or an amount or conditions as prescribed	5%	
	(b)	Immovable property, preference and ordinary shares in property companies, and linked units comprising shares linked to debentures in property companies, not listed on an exchange	5%	15%
5.	COMMODITIES			10%
5.1	Inside the Republic and foreign assets			
	(a)	Kruger Rands and other commodities listed on an exchange, including exchange traded commodities: -		10%
	(i)	gold	10%	
	(ii)	each other commodity	5%	
6.	INVESTMENTS IN THE BUSINESS OF A PARTICIPATING EMPLOYER INSIDE THE REPUBLIC IN TERMS OF: -			
	(a)	section 19(4) of the Pension Funds Act		5%
	(b)	To the extent it has been allowed by an exemption in terms of section 19(4A) of the Pension Funds Act		10%
7.	HOUSING LOANS GRANTED TO MEMBERS IN ACCORDANCE WITH THE PROVISIONS OF SECTION 19(5)			95%

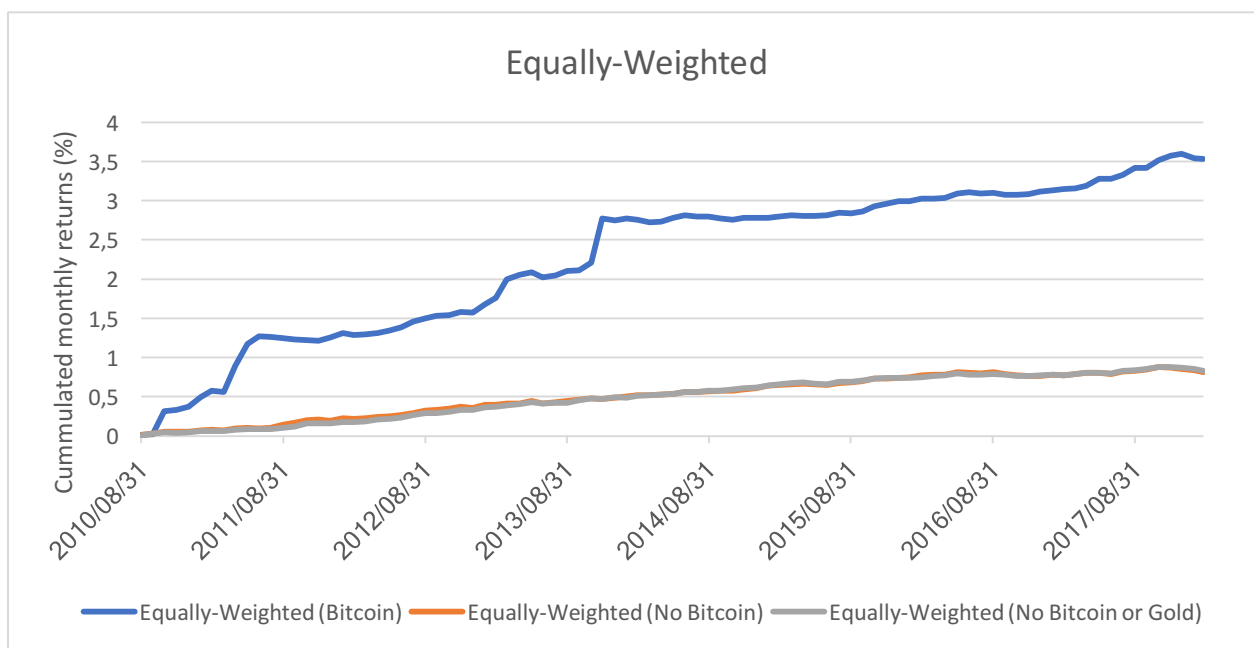
8.	HEDGE FUNDS, PRIVATE EQUITY FUNDS AND ANY OTHER ASSET NOT REFERRED TO IN THIS SCHEDULE			15%
8.1	Inside the Republic and foreign assets			
	(a)	Hedge funds		10%
		(i)	Funds of hedge funds	5% per fund of hedge funds
		(ii)	Hedge funds	2.5% per hedge fund
	(b)	Private equity funds		10%
		(i)	Funds of private equity funds	5% per fund of private equity funds
		(ii)	Private equity funds	2.5% per private equity fund
	(c)	Other assets not referred to in this schedule and excluding a hedge fund or private equity fund		2.5%

Source: Momentum

<http://content.momentum.co.za/Content/regulation-28/reg-28-budget-2011.pdf>

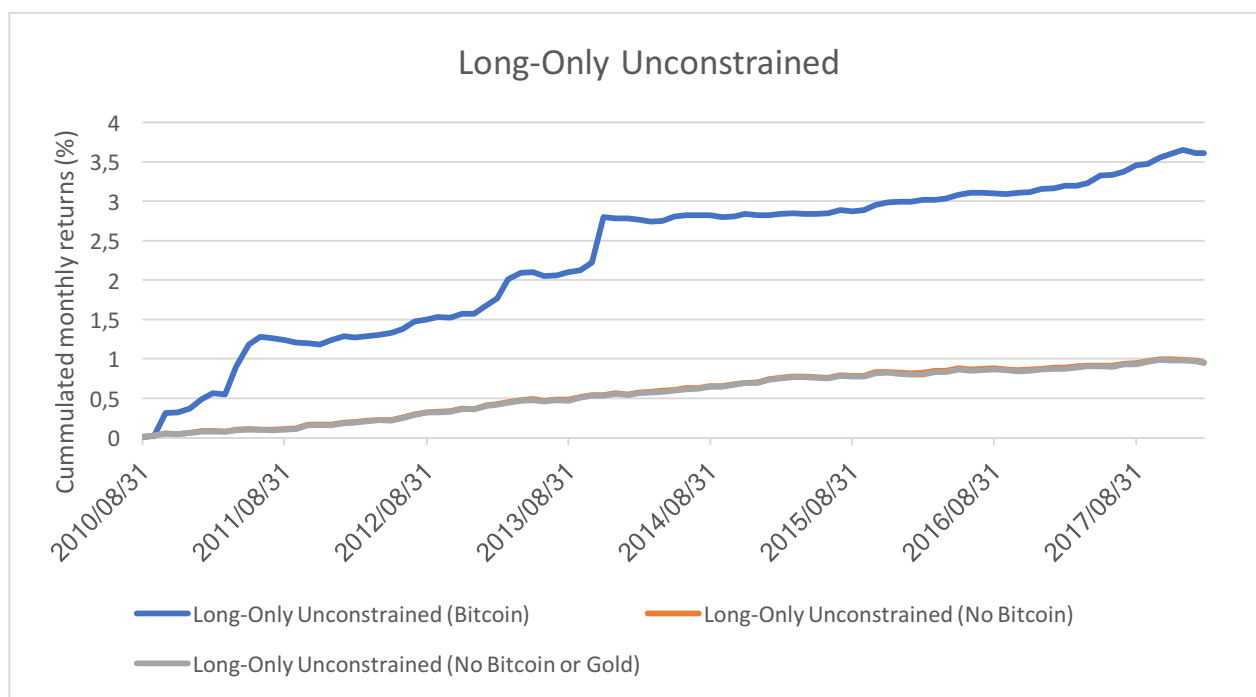
Appendix A1.15: Equally-weighted portfolios cumulated returns

This figure displays the cumulated returns of the three variations of equally-weighted optimal portfolios over the investment period. Returns are kept at constant optimal weighting over the investment period, and re-weighted every month.



Appendix A1.16: Long-only unconstrained portfolios cumulated returns

This figure displays the cumulated returns of the three variations of long-only optimal portfolios over the investment period. Returns are kept at constant optimal weighting over the investment period, and re-weighted every month.



Appendix A1.17: +100%/-100% portfolios cumulated returns

This figure displays the cumulated returns of the three variations of +100%/-100% optimal portfolios over the investment period. Returns are kept at constant optimal weighting over the investment period, and re-weighted every month.

